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"Spreading E-Mobility Everywhere"

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Université de Valenciennes et du Hainaut-Cambrésis

LAMIH LABORATOIRE D'AUTOMATIQUE DE MECANIQUE ET D'INFORMATIQUE INDUSTRIELLES ET HUMAINES UMR CNRS 8201

MEGEVH French network on HEV's

Optimal control of hybrid powertrain with the Pontryagin Minimum Principle

Part 1 – Optimal control

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France

Summary

Part 1 : Optimal control

I) Introduction

II) Hybrid vehicle

- Input&Output
- Quasi-static modeling

III) Optimal control of hybrid vehicle

- Energy management as an optimal control problem
- Pontryagin Minimum Principle
  - Basic Algorithm
  - Mathematical models
  - Binary & integer variable optimization
  - State Constraints

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Introduction

Definition:
A hybrid vehicle uses at least two energy sources for its propelling, one of them must be reversible

Some energy sources & storage

- Electric motor & batteries :

/Toyota/

- Kinetic energy recovery system (KERS)
  - Flywheel+CVT
  - Flywheel + Electric machines

/Porsche/

- Pneumatic
  - Compressor + Pneumatic engine

/Peugeot/

Introduction

All these technologies add one or more degree of freedom in the powertrain control that can be used to increase:

- Dynamical performances (0-100km/h)
- Driving comfort
- Mileage & pollution reduction

<u>One degree of freedom :</u>

- A supervisory control algorithm is necessary
- Poor control can be less efficient than conventional car

=> Proposed approach : optimization & optimal control theory

Acknowledgments

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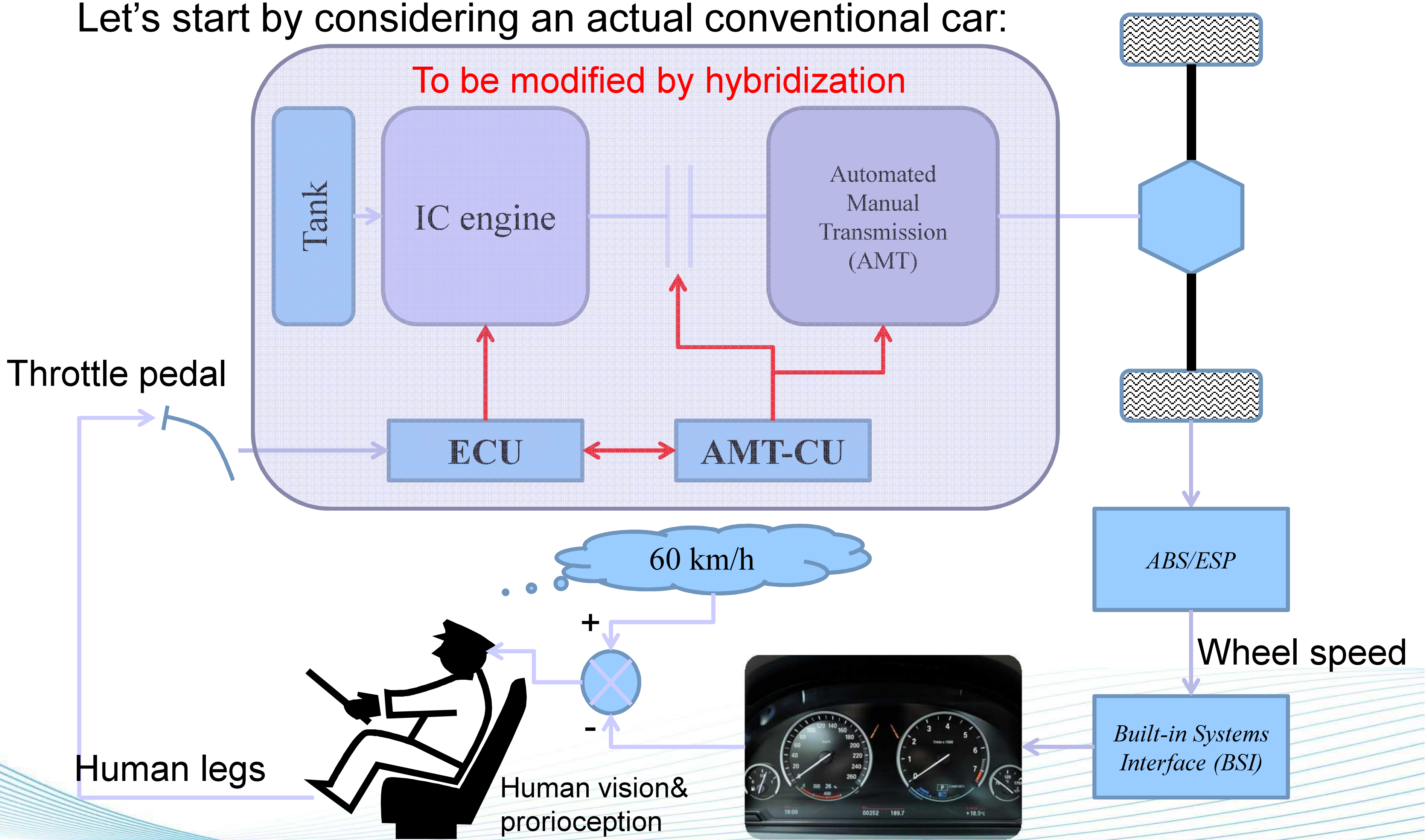
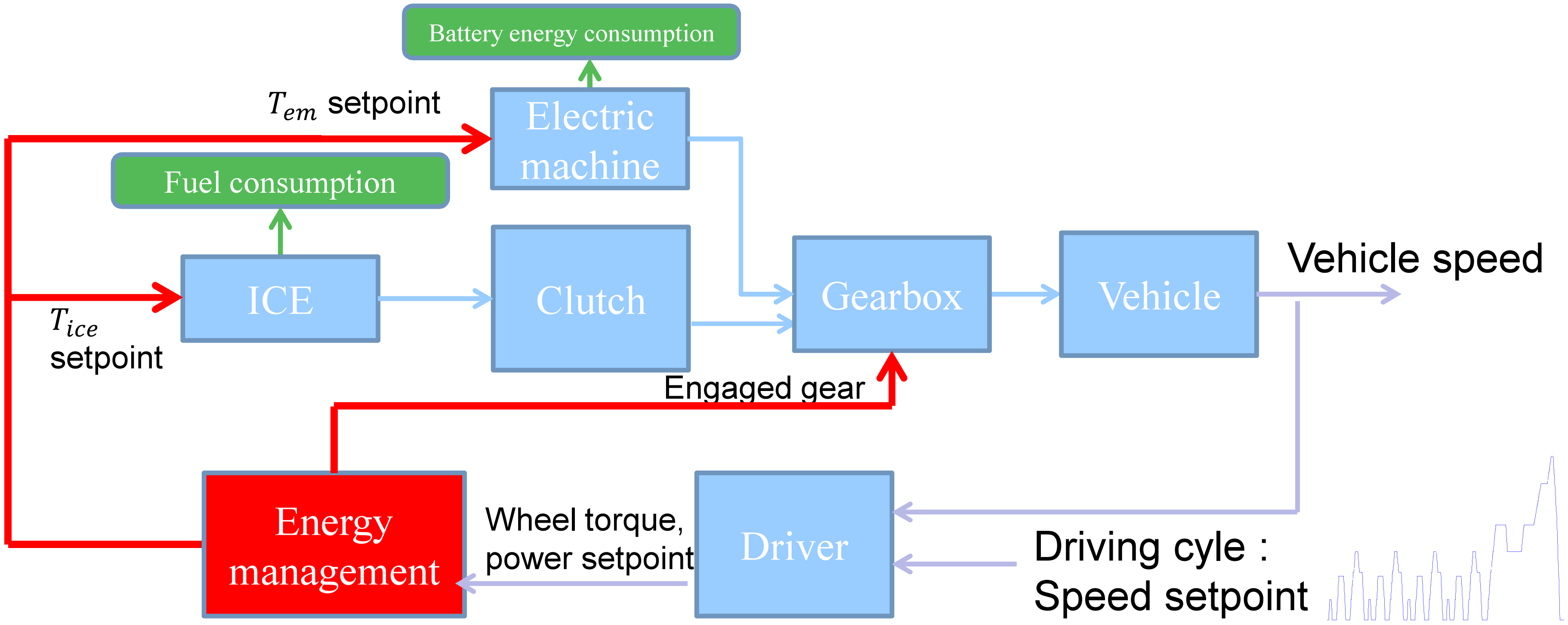
Hybrid vehicle

Let's start by considering an actual conventional car:



Hybrid vehicle

“Forward” modeling approach



[Actual vehicle :] “Forward” behavior

Speed is more or less controlled by a “driver”

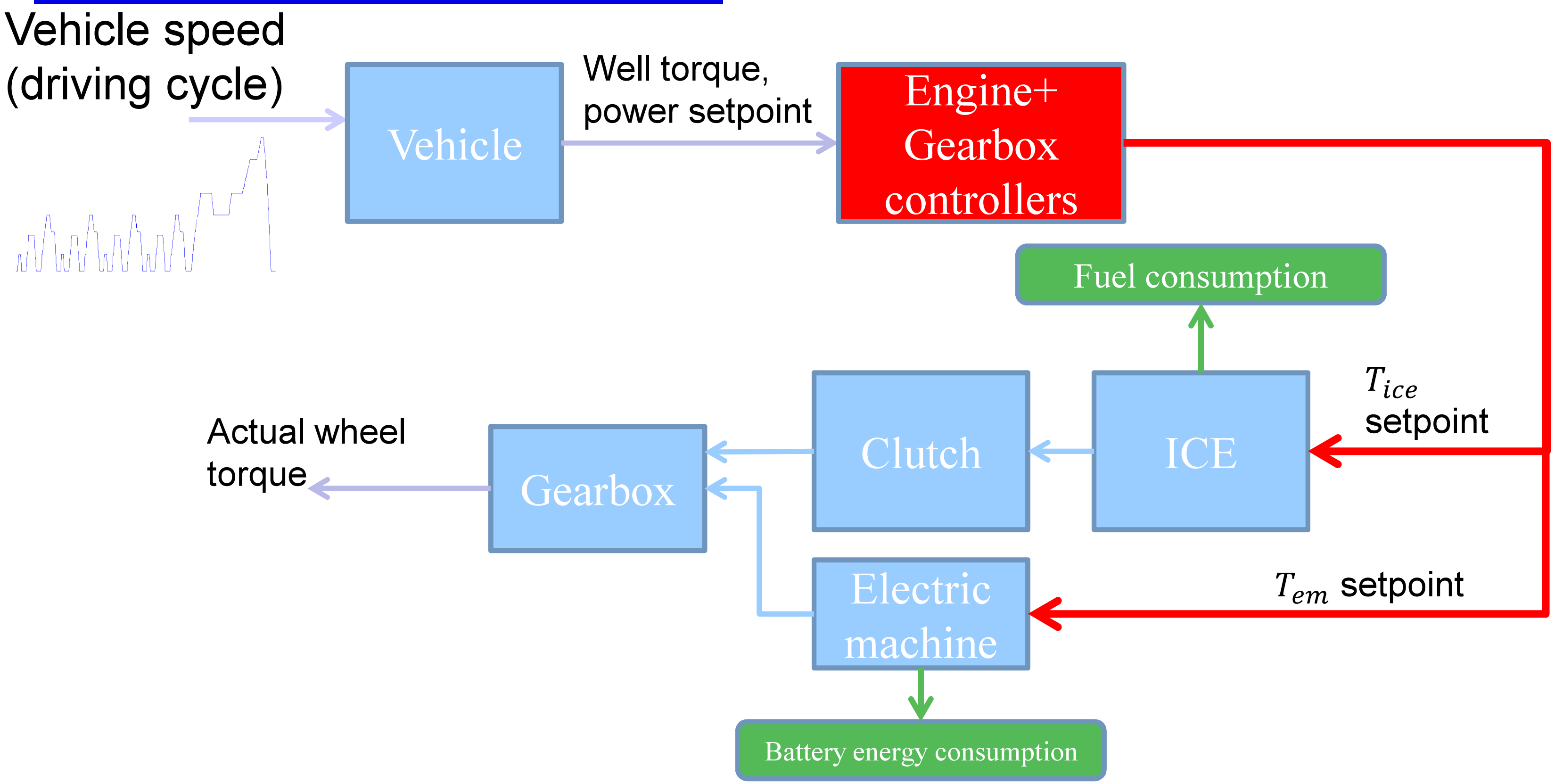
Throttle pedal = torque at the wheel setpoint.

Torque is the input, speed the output

A “driver” is necessary : impact on the fuel consumption ?

Hybrid vehicle

"Backward" modeling approach



Inverse dynamics problem:

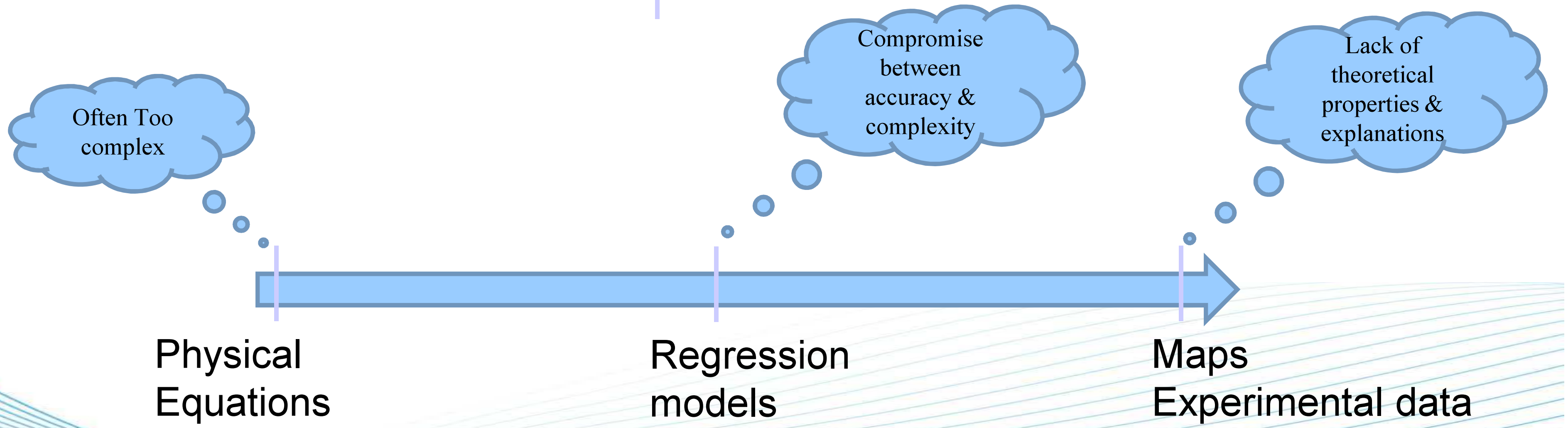
- *The vehicle follows exactly the driving cycle*
- *No driver*
- *Must keep controller with physical inputs (i.e. torque input)*
- *Controller should only use available signals !*

Quasi-static modeling

Energetic behaviour:
- Low bandwitdh : 0-5Hz typical, transients are not so important
- Nonlinearities are fundamental

A few dynamical models:
- Vehicle
- Energy storage

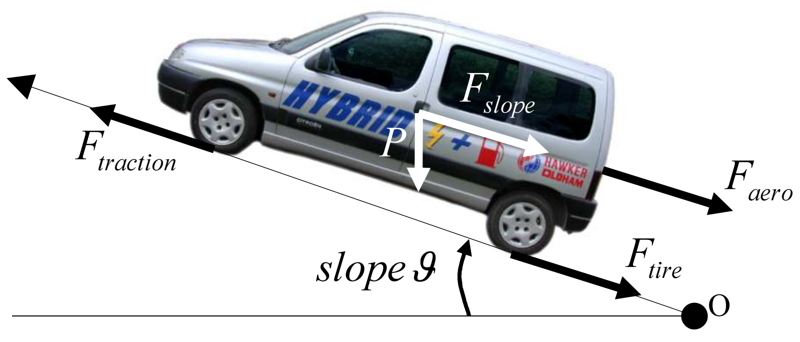
Many static components:
- Engine
- Electric machines
- Gears
- Clutches



Quasi-static modeling

<u>Vehicle dynamics :</u>

- Quarter vehicle mode
- No tire slip modeling but rolling resistance



$$M_{total} \cdot \frac{dV_{veh}}{dt}(t) = F_{traction}(t) - F_{tire}(t) - F_{slope}(t) - F_{aero}(t)$$

$$M_{total} = M_{veh} + M_{eq}$$

Equivalent mass of rotating inertias

Vehicle mass

$$F_{tire}(t) = c_r\left(v(t), p(t), \dots\right) \cdot \underbrace{M_{veh} \cdot g \cdot \cos(\vartheta)}_{\text{Tire Normal Load}}$$

$$F_{aero}(t) = \frac{1}{2} \cdot \rho_{air} \cdot S_f \cdot C_d \cdot V_{veh}(t)^2$$

$$F_{slope}(t) = M_{VEH} \cdot g \cdot \sin\left(\vartheta(t)\right)$$

$$F_{traction}(t) = T_{traction}(t) / r_{tire}$$

r_{tire} Tire radius

$T_{traction}$ Powertrain torque

Quasi-static modeling

<u>Clutch modeling:</u>

- Clutch slipping is complex (Friction, Stiction, Wear)
- Control is driving by comfort issues
- Slipping phases are limited (T° rise)

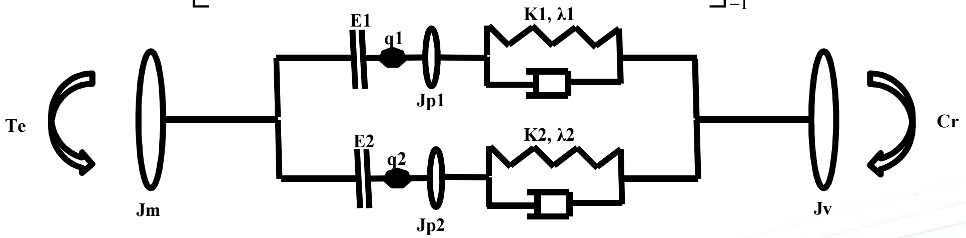
=> No need for a complex clutch model

$$J_m\dot{\omega}_m = T_e - C_{e1} - C_{e2} - f_m\omega_m$$

$$J_{p1,2}\dot{\omega}_{p1,2} = C_{e1,2} - \left[k_{1,2}\int\left(q_{1,2}\omega_{p1,2} - \omega_v\right) - \lambda_{1,2}\left(q_{1,2}\omega_{p1,2} - \omega_v\right)\right]q_{1,2} - f_{p1,2}\omega_{p1,2}$$

$$J_v\dot{\omega}_v = \left[k_{1,2}\int\left(q_{1,2}\omega_{p1,2} - \omega_v\right) - \lambda_{1,2}\left(q_{1,2}\omega_{p1,2} - \omega_v\right)\right] - f_v\omega_v - C_r$$

$$C_{e1,2} = \left[\frac{\lambda_{e1,2}\left(\omega_m - \omega_{p1,2}\right) + k_{e1,2}\alpha_{g1,2}}{Cc_{1,2}(x_{1,2})\varepsilon_{1,2}}\right]_{-1}^{+1} Cc_{1,2}(x_{1,2})\varepsilon_{1,2}$$



Quasi-static modeling

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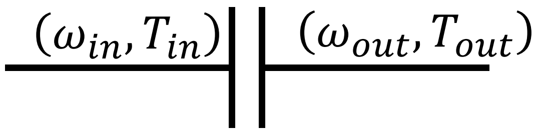

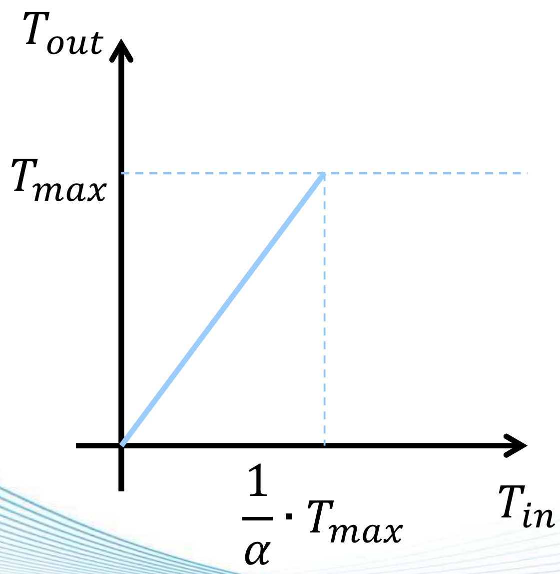

<u>Slipping phase:</u>

$$\alpha = \frac{\omega_{idle\_speed}}{\omega_{out}}$$

$$T_{out} = \alpha \cdot T_{out}$$

Quasi-static modeling

Engine modeling:
- Fundamental for good energetic performance measure
- Static is often ok, air path dynamics may increase accuracy
- Take care of pumping & friction losses (negative torque, fuel injection cutoff)
- T° : fuel consumption depends on T°. Measurements ? => simple correction scheme

Equation from physics are not usable (combustion => finite elements)

In practice :
- Lookup table data : fuel consumption = f(Torque,Rotational Speed)

To measure a map :
- IC Engine test bench
- Vehicle additional sensors

Quasi-static modeling

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To measure a map :

- Vehicle & additional sensors

Side shaft strain gauges:

Fuel flow meter :
(+ T° & Pressure)

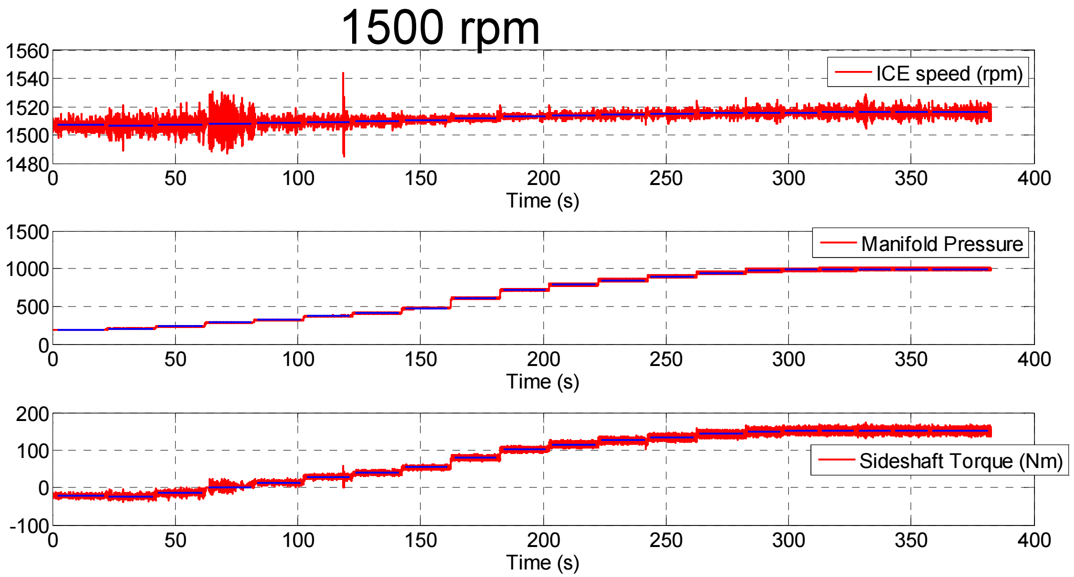
Manifold pressure, IC engine, Temperature, speed :
=> OBD and/or CAN

Roller bench
(3-4 hours)

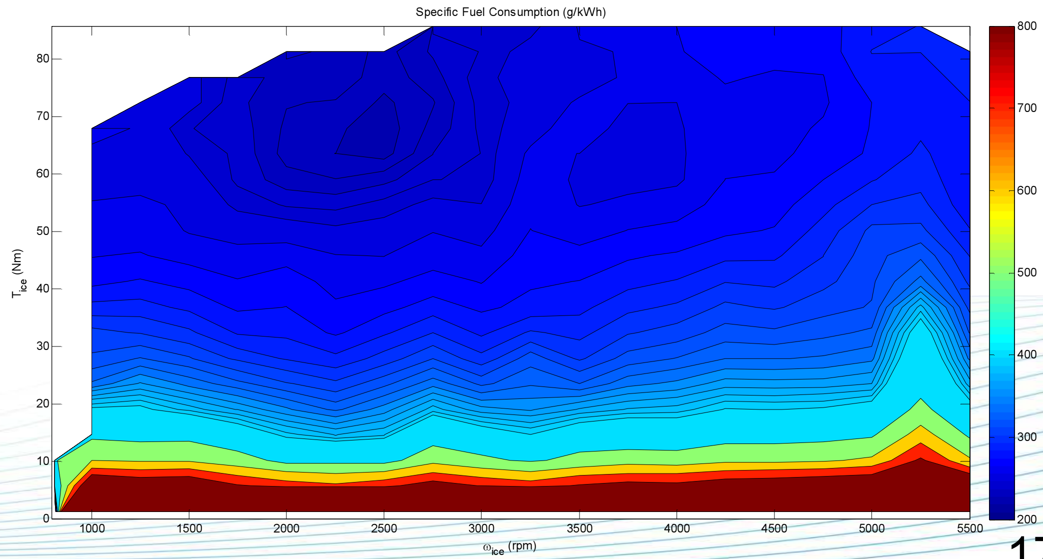
Quasi-static modeling

To measure a map :

- Vehicle & additional sensors



Repeat for all speeds
(e.g. step 250 rpm)

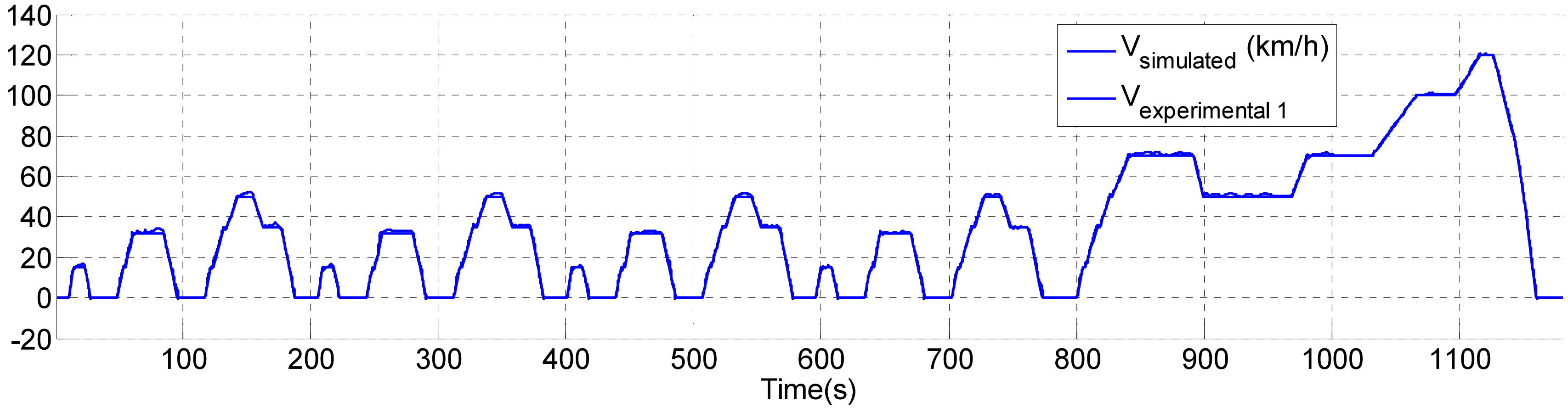


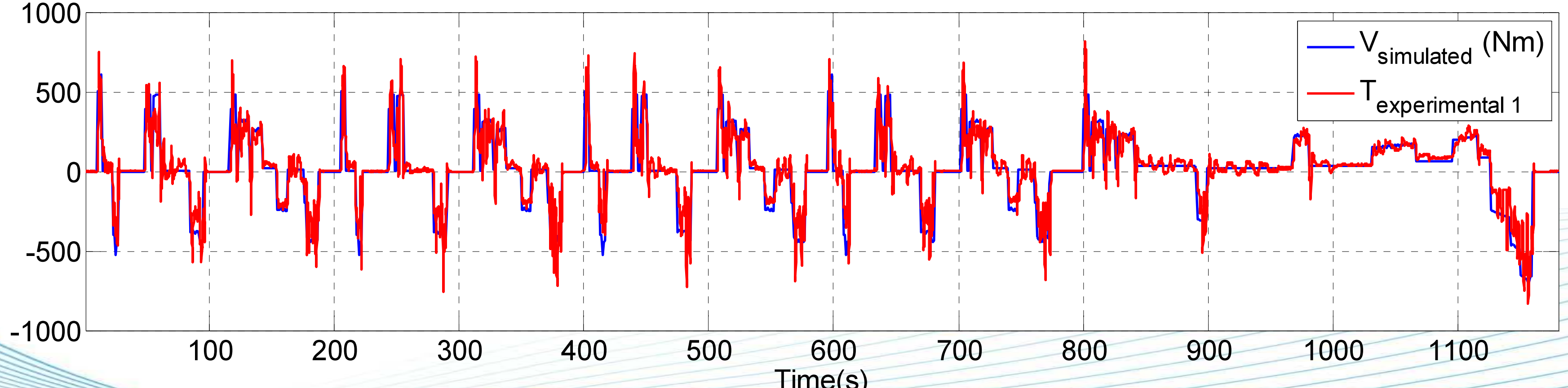
Quasi-static modeling

Fuel consumption :

- Experiment (hot conditions) : 6,95 l/100 km
- Simulation : 7,09 l/100 km

2% error

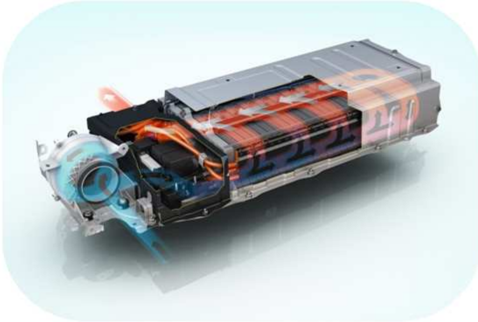




Quasi-static modeling

Energy storage

Quasi static energetic modeling ≠ electrical circuit modeling



/Toyota/

- Battery

State of charge : $Soc(t) = \frac{-1}{Q_{ah} \cdot 3600} \int_0^t I(\tau) \cdot d\tau$

Constraints :

- State of charge constraints: $Soc(t) \in \left[\underline{Soc}, \overline{Soc} \right]$
- Voltage constraint: $V(t) \in \left[\underline{V}, \overline{V} \right]$

/Maxwell/

- Supercapacitor

Voltage: $U(t) = \frac{-1}{C} \int_0^t I(\tau) \cdot d\tau$

Energy : $E(t) = \frac{1}{2} \cdot C \cdot U(t)^2$

Constraints :

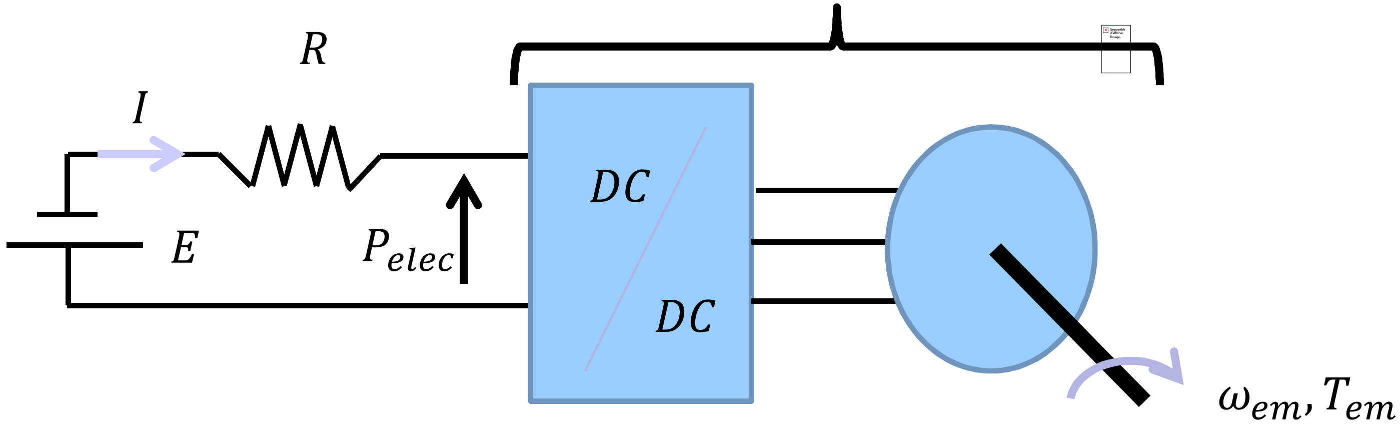
- Voltage: $V(t) \in \left[\underline{V}, \overline{V} \right]$

Quasi-static modeling

Electric machine + Power converters:

- No noticeable dynamics (Typ. torque response time <10ms vs 300ms for ICE)
- Dependence on T° : torque limitation
- Losses depends on energy storage voltage (therefore on Soc/Soe)
- *Need to include the energy storage electrical model*

$$PowerLosses = f(\omega_{em}, T_{em}, \dots)$$



$$P_{elec} = \omega_{em} \cdot T_{em} + P_{losses}(\cdot)$$

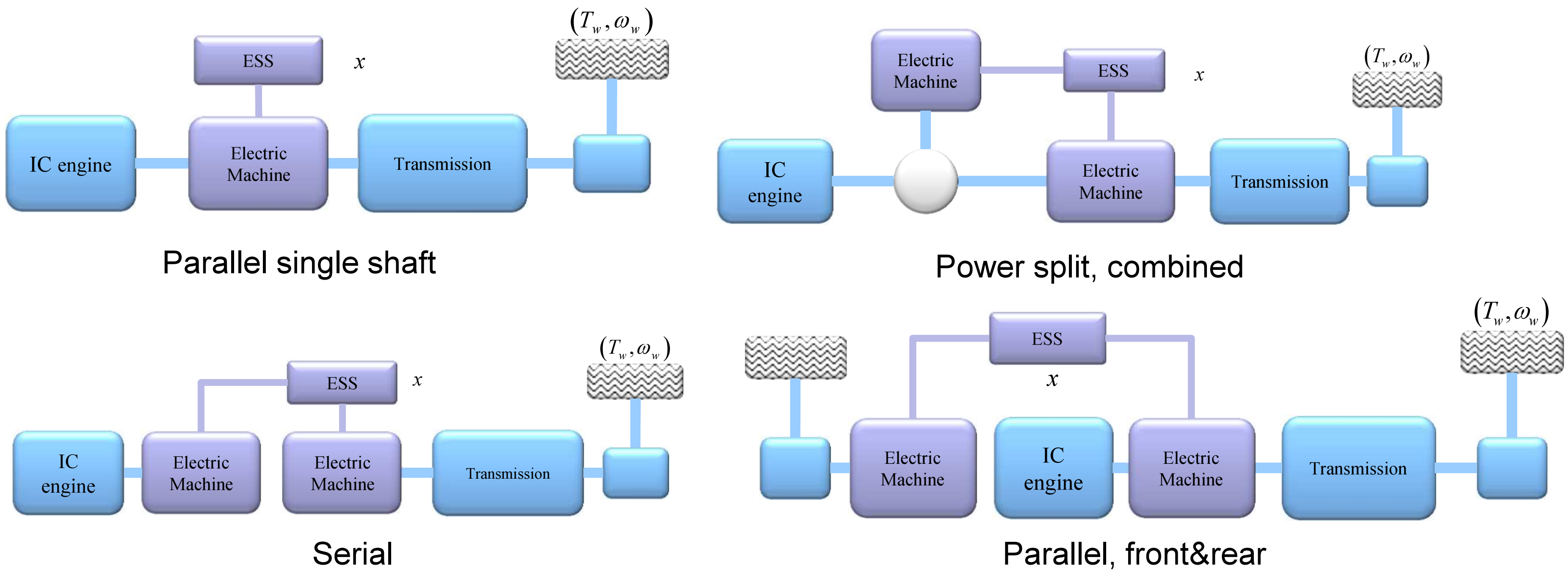
$$P_{elec} = E(Soc) \cdot I - R(Soc) \cdot I^2$$

Finally :

$$I = f(\omega_{em}, T_{em}, Soc)$$

Quasi-static modeling

Hybrid vehicles arrangement:



Wide range of vehicle architectures & sizing

⇨ Generic optimization problem ??

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HEV energy management

<u>Energy management algorithm objectives:</u>

1) Fuel consumption minimization

Emissions reduction?

No simple model available for the ICE exhaust after treatment system

Rely on ECU combustion management

Ensure good temperature for better emission control

Static conditions : mixed fuel consumption & emission map

=> same optimization problem

2) What about second energy source ?

- Considering only one energy source leads to trivial solutions

Fuel consumption minimization => pure electric mode

Battery recharge maximization => max IC power

2 cases : real time & simulation

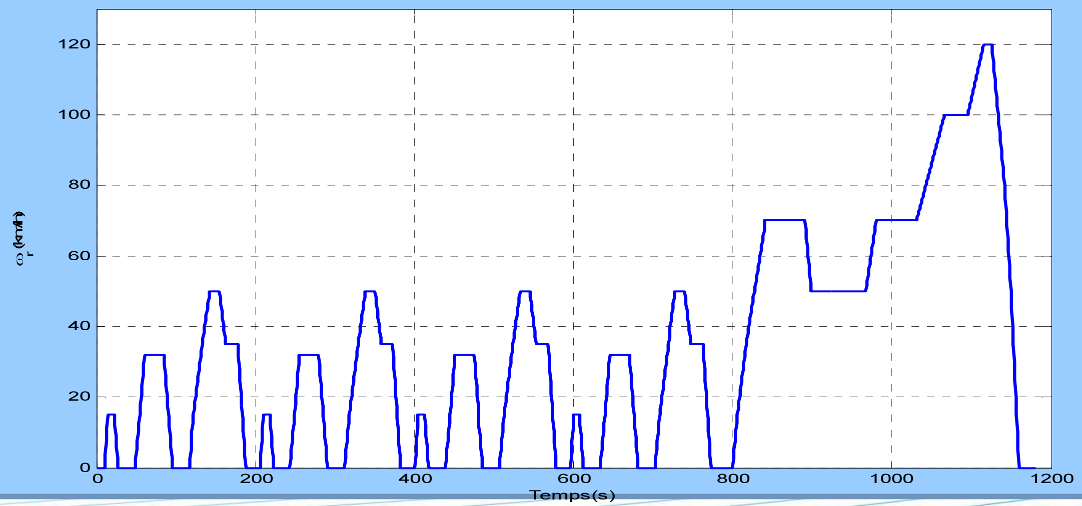
HEV energy management

Real time driving conditions:

- Driving conditions are unknown : trip length, urban or highway ?
- Driver may drive nervously or can be very calm
- Sub-optimal control

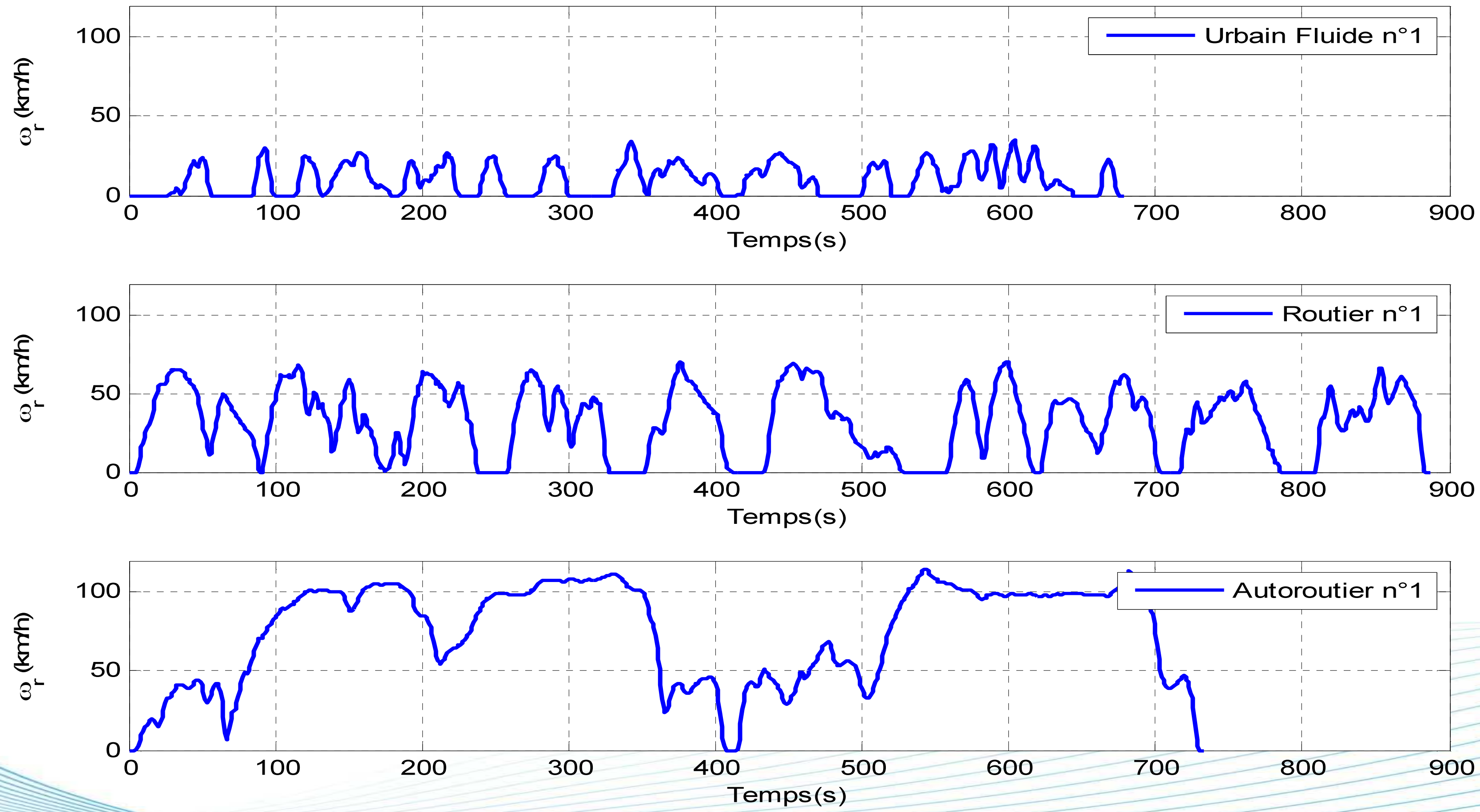
Simulation:

- Need reproducible experimentations
- Driving cycle is fixed and can be supposed known
- Optimal control



HEV energy management

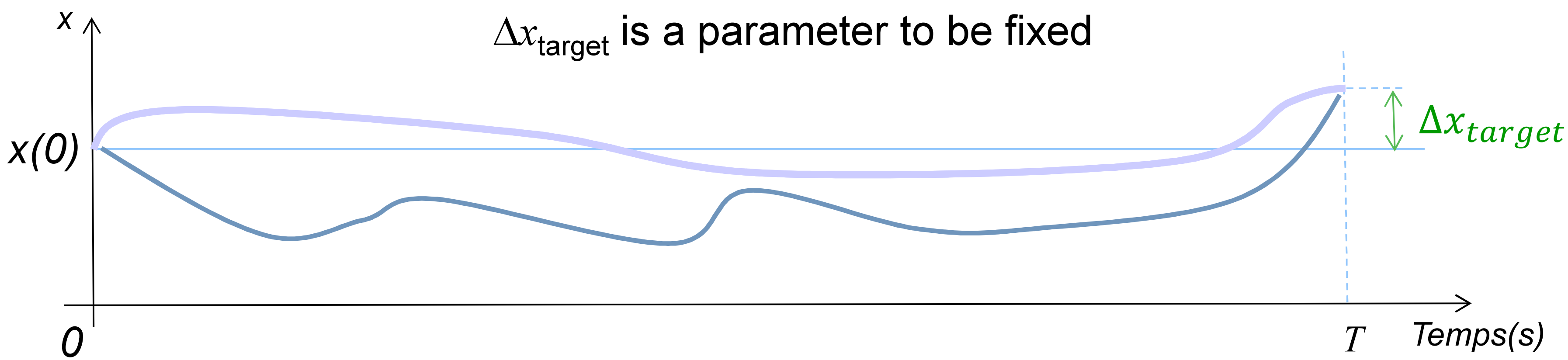
A few realistic driving cycles from the Hyzem or Artemis studies:



HEV energy management

Energy Storage System (ESS) state management:

- Simulation : No a priori on the best ESS usage



If Δx_{target} =0 then :

- Propulsion energy is only provided by fuel
- ESS is used as a temporary energy storage
- Fuel consumption can be compared with conventional car

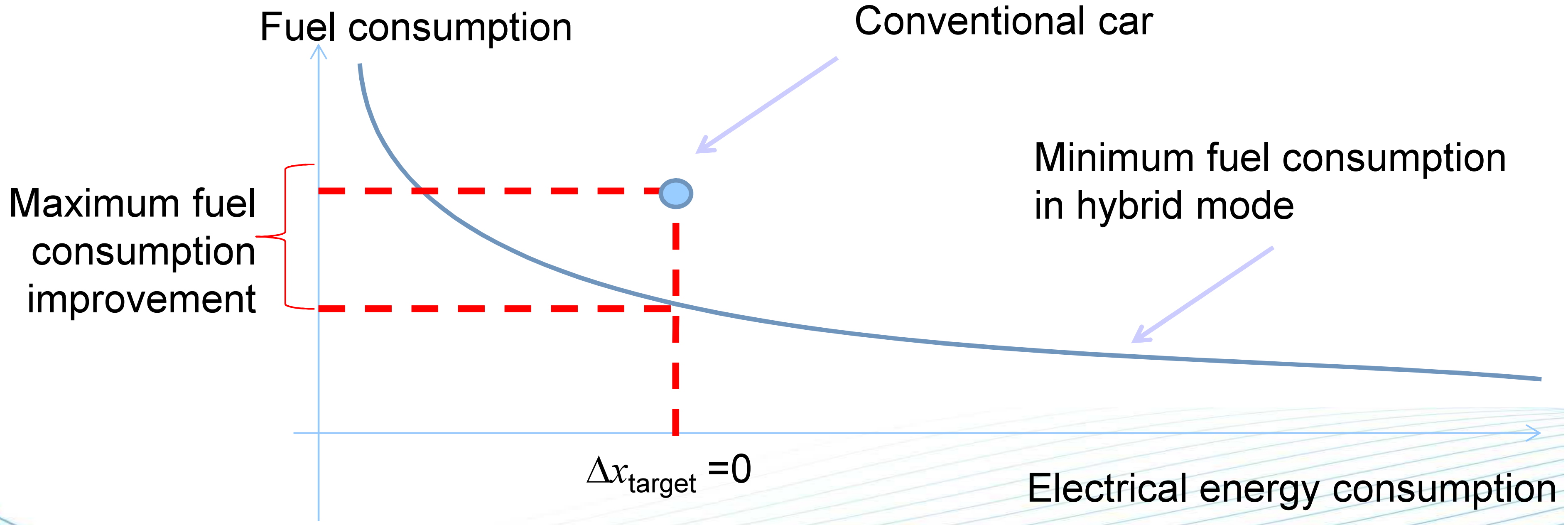
In general, the optimal state trajectory can be whatever is needed to optimize the fuel consumption

HEV energy management

Energy Storage System (ESS) state management:

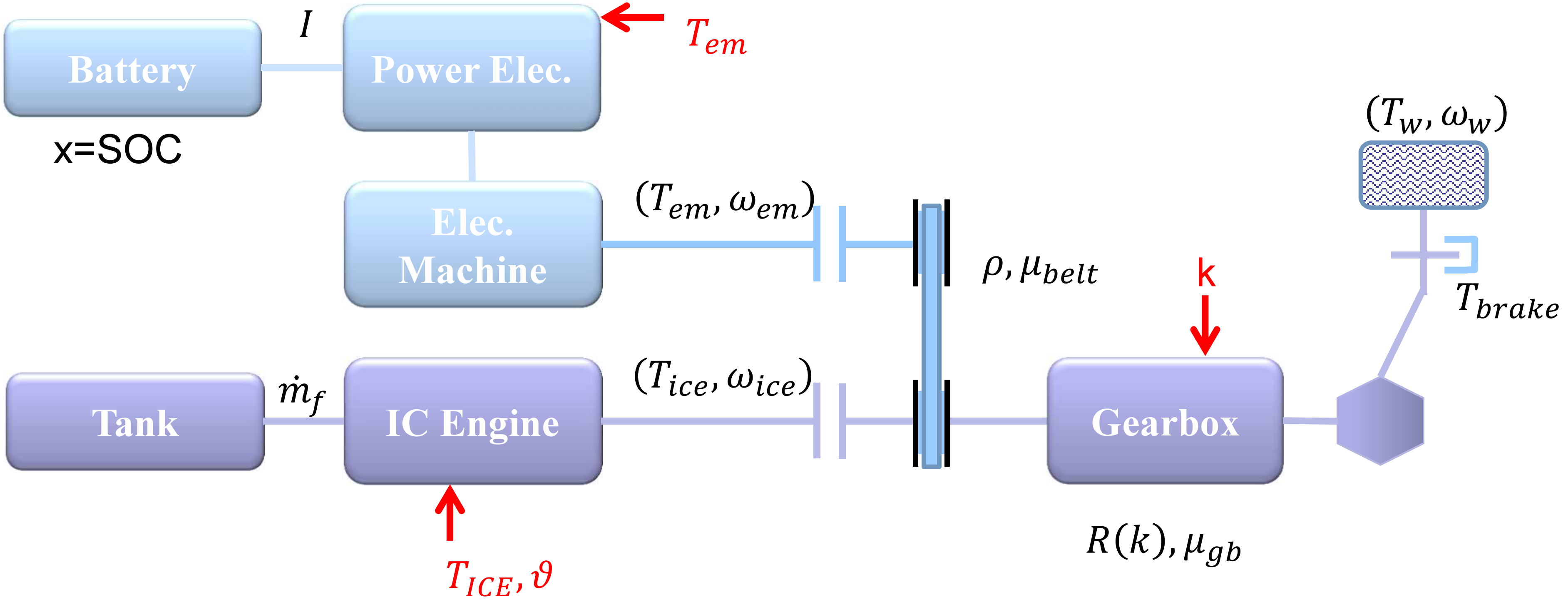
- Simulation : Why optimal control is important ?

By computing the optimal state trajectory for different Δx_{target} values, a « global » analysis of the energetic performance can be obtained.



HEV energy management

Case study: Parallel single shaft vehicle, Full Hybrid



1 arrangement = 1 set of algebraic constraints :

$$\omega_w(t) = \frac{\omega_{ice}(t)}{R(k(t))} = \frac{\omega_{em}(t)}{R(k(t)) \cdot \rho}$$

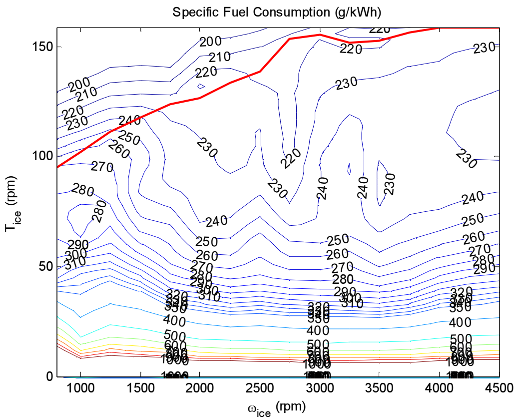
$$T_w(t) = \mu_{gb}^{\,sign(T_w(t))} \cdot R(k(t)) \cdot \left(T_{ice}(t) \cdot \vartheta(t) + \rho \cdot \mu^{\,sign(T_{em}(t))} \cdot T_{em}(t)\right)$$

$$T_w(t) = R(k(t)) \cdot \left(T_{ice}(t) \cdot \vartheta(t) + \rho \cdot T_{em}(t)\right)$$

HEV energy management

Model parameters:

- *Vehicle : Peugeot 308 SW* 1837 kg. including battery
- *Electric machine : 30kW*
- *ICE : 76kW EP6*
- *Battery Li-ion: 6,4 Ah, 320V, 55 kW*



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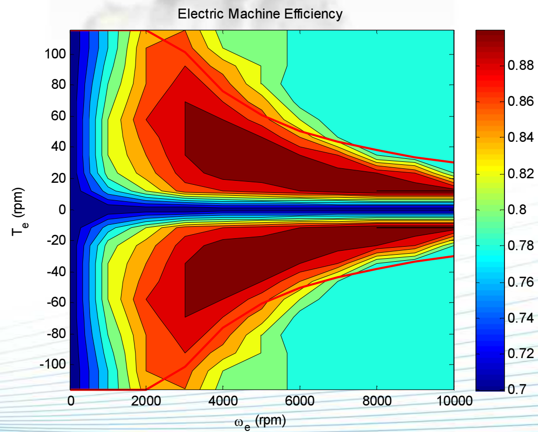
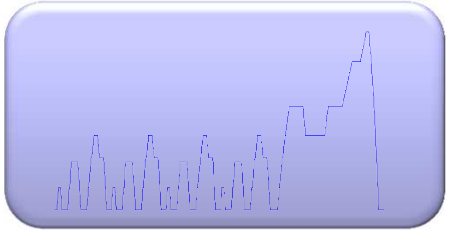


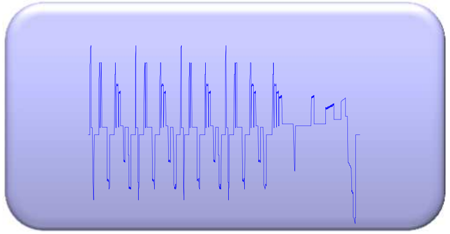
HEV energy management

<u>*Input data:*</u>



Driving cycle $\omega_w(t)$

Inverse dynamics
Vehicle model



Torque at the wheel $T_w(t)$

<u>*Step 1 : List all the equations*</u>

Fuel consumption:

$$J = \int_0^T \dot{m}_f\left(\omega_{ice}(t), T_{ice}(t)\right) \cdot \vartheta(t) \cdot dt$$

System dynamics:

$$\dot{x}(t) = \frac{-1}{Q_{ah} \cdot 3600} I\left(\omega_{em}(t), T_{em}(t), x(t)\right)$$

Constraints:

$$T_w(t) = R\left(k(t)\right) \cdot \left(T_{ice}(t) \cdot \vartheta(t) + \rho \cdot T_{em}(t)\right)$$

$$\omega_w(t) = \frac{\omega_{ice}(t)}{R\left(k(t)\right)} = \frac{\omega_{em}(t)}{R\left(k(t)\right) \cdot \rho}$$

$$T_{ice}(t) \in \left[\underline{T_{ice}}(t), \overline{T_{ice}}(t)\right]$$

$$T_{em}(t) \in \left[\underline{T_{em}}(t), \overline{T_{em}}(t)\right]$$

$$x(t) \in \left[\underline{x}(t), \overline{x}(t)\right]$$

HEV management

Step 2 : Choice of the decision variables

Discrete decision variables :

- IC engine fuel injection : on/off $\vartheta(t) \in \{0,1\}$
- Engaged gear : $k(t) \in \{0,..,n_{gb}\}$

Continuous decision variables: $T_w(t) = R(k(t)) \cdot (T_{ice}(t) \cdot \vartheta(t) + \rho \cdot T_{em}(t))$

2 variables, 1 algebraic constraint => 1 degree of freedom

Some possibilities:

- T_{ice} or T_{em}
- Fuel mass flow $(\dot{m}_f)$ if $T_{ice} = f(\dot{m}_f, \cdot)$ exists and well defined (Willans model)
- Energy storage current (I) if $T_{em} = f(\mathrm{I}, \cdot)$ exists and well defined

Finally:

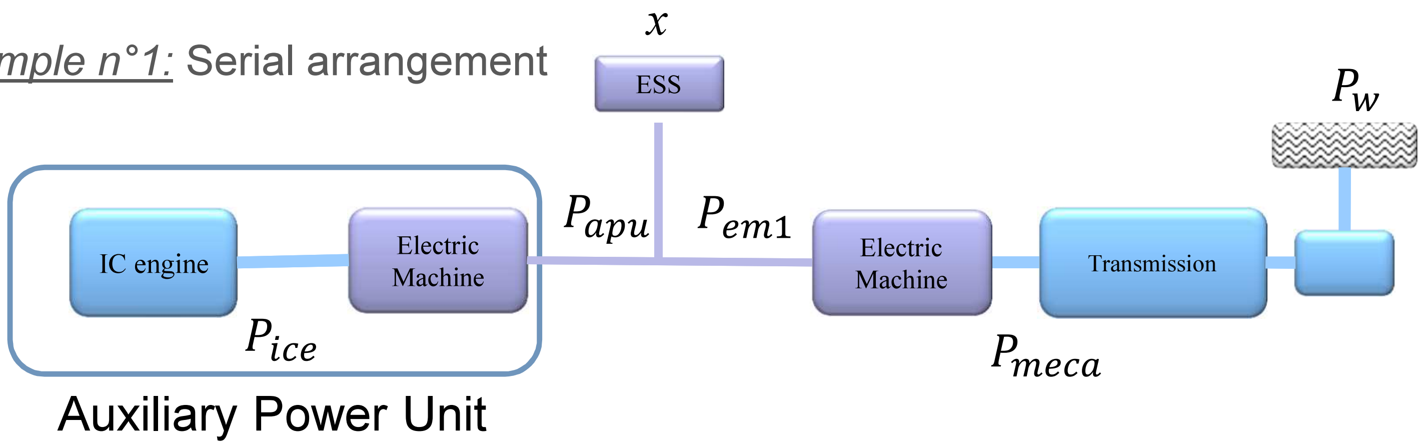
$$u(t) = \begin{bmatrix} u_c(t) & u_d(t) \end{bmatrix}^T \qquad \begin{cases} u_d(t) = \begin{bmatrix} \vartheta(t) & k(t) \end{bmatrix}^T \\ u_c(t) = T_{ice}(t) \end{cases}$$

HEV management

<u>Step 3 :</u> Identification of the exogenous variables $W(t)$:

Exogenous variables: variable that affect the energetic model, that cannot be controlled *and that is not affected by the energetic model*.

<u>Example n°1:</u> Serial arrangement



<u>Exogenous variable choice depend on bus voltage:</u>

- Constant bus voltage : $P_{em1} = f(P_{meca})$ => $W(t) = P_{em1}(t)$
- Variable bus voltage $P_{em1} = f(P_{meca}, x)$ => $W(t) = P_{meca}(t)$

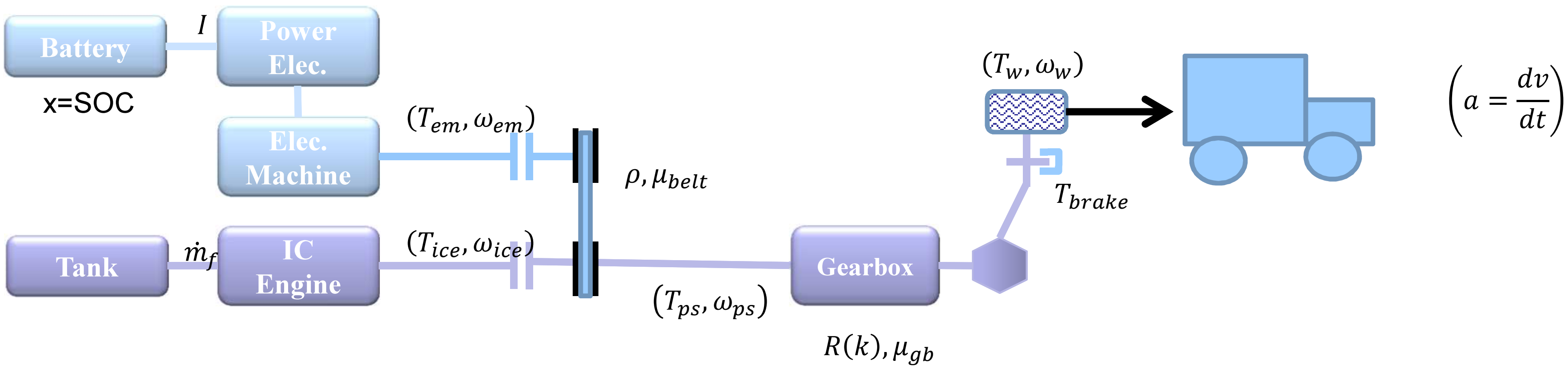
Depends on state

HEV management

<u>Step 3 :</u> Identification of the exogenous variables $W(t)$:

Exogenous variables: variable that affect the energetic model, that cannot be controlled *and that is not affected by the energetic model*.

<u>Example n°2:</u> Parallel single shaft arrangement



<u>Exogenous variable choice depend whether or not the engaged gear is optimized:</u>

$Vehcule\ model$: $\omega_{ps} = v \cdot \frac{R(k)}{R_{tire}} \omega_w$ and $\frac{dv}{dt} = \left(M + J_{ice} \cdot \frac{R(k)^2}{R_{tire}^2}\right)^{-1} \cdot \left(\frac{R(k)}{R_{tire}} \cdot T_{ps} - F_{res}\right)$

- If k is fixed : $W(t) = [T_{ps}(t) \quad \omega_{ps}(t)]^T$
- f k is optimized : $W(t) = \left[\frac{dv}{dt}(t) \quad v(t)\right]^T$

HEV management

Step 4 : Sets of possible values of the decisions variables

- IC engine fuel injection : on/off $\vartheta(t) \in \{0,1\}$

Pure electric mode possible if $T_w(t) \leq R(k) \cdot \overline{T_{em}}(\omega_w(t) \cdot R(k) \cdot \rho)$

$\Rightarrow$ The set of admissible values is $\Phi_\vartheta(W(t))$

$$\vartheta(t) \in \Phi_\vartheta(W(t)) \subset \{\{0\},\{1\},\{0,1\}\}$$

Pure electric mode only:

ICE can not be used.

$$\Phi_\vartheta(W(t)) = \{0\}$$

E.g. : Clutch slipping forbidden

Forced Hybrid mode:

ICE is necessary on.

$$\Phi_\vartheta(W(t)) = \{1\}$$

E.g. high rotational speed, depending on the EM gear set ratio

Non constrained mode:

ICE can be on or off

$$\Phi_\vartheta(W(t)) = \{0,1\}$$

HEV management

<u>Step 4 :</u> Sets of possible values of the decisions variables

<u>Discrete decision variables :</u>

- Engaged gear : $k(t) \in \{0, .., n_{gb}\}$

A gear can be selected if :

- Speed constraint is ok : $\omega_w(t) \cdot R(k) \leq \overline{\omega_{ice}}$
- Torque constraint is ok: $\frac{T_w(t)}{R(k)} \leq \overline{T_{ice}}(\omega_w(t) \cdot R(k)) + \overline{T_{em}}(\omega_w(t) \cdot R(k) \cdot \rho)$

As a result, the set of admissible values for the engaged gear is defined:

$$k \in \Phi_k(W(t))$$

HEV management

Step 4 : Sets of possible values of the decisions variables

Discrete decision variables :

Combining both sets, the set of admissible values for discrete variable is defined

$$\vartheta(t) \in \Phi_\vartheta\big(W(t)\big) \subset \big\{\{0\},\{1\},\{0,1\}\big\} \quad \text{and} \qquad k \in \Phi_k\big(W(t)\big)$$

$$\Rightarrow u_d(t) = [\vartheta(t) \quad k(t)]^T \in \Phi_d\big(W(t)\big)$$

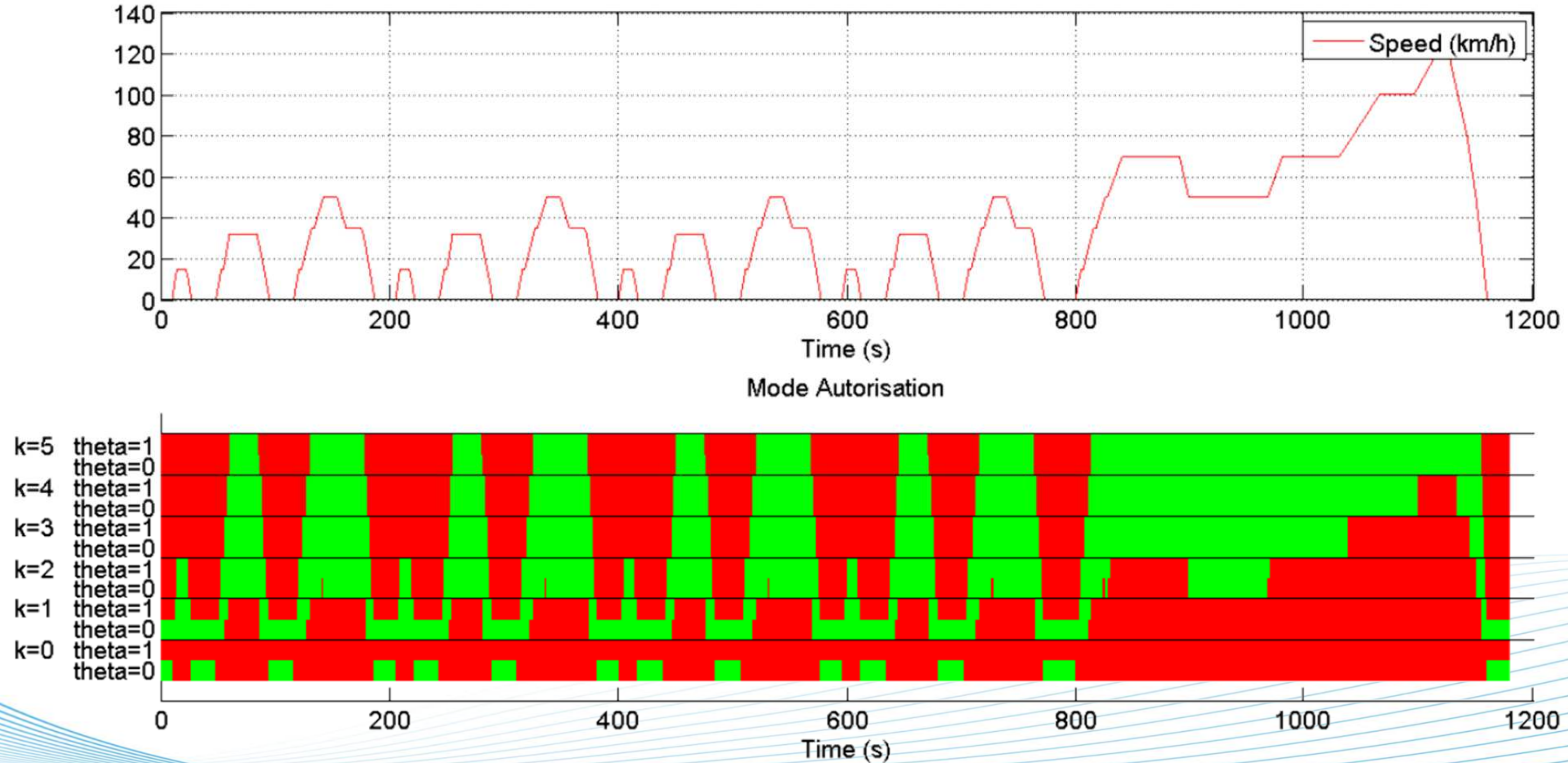
With $\Phi_d\big(W(t)\big) = \Phi_\vartheta\big(W(t)\big) \times \Phi_k\big(W(t)\big)$

Most of the programs need to start with the computation of this set

HEV management

<u>Step 4 :</u> Sets of possible values of the decisions variables

In practice, available gear should be limited to avoid driving comfort issues



HEV management

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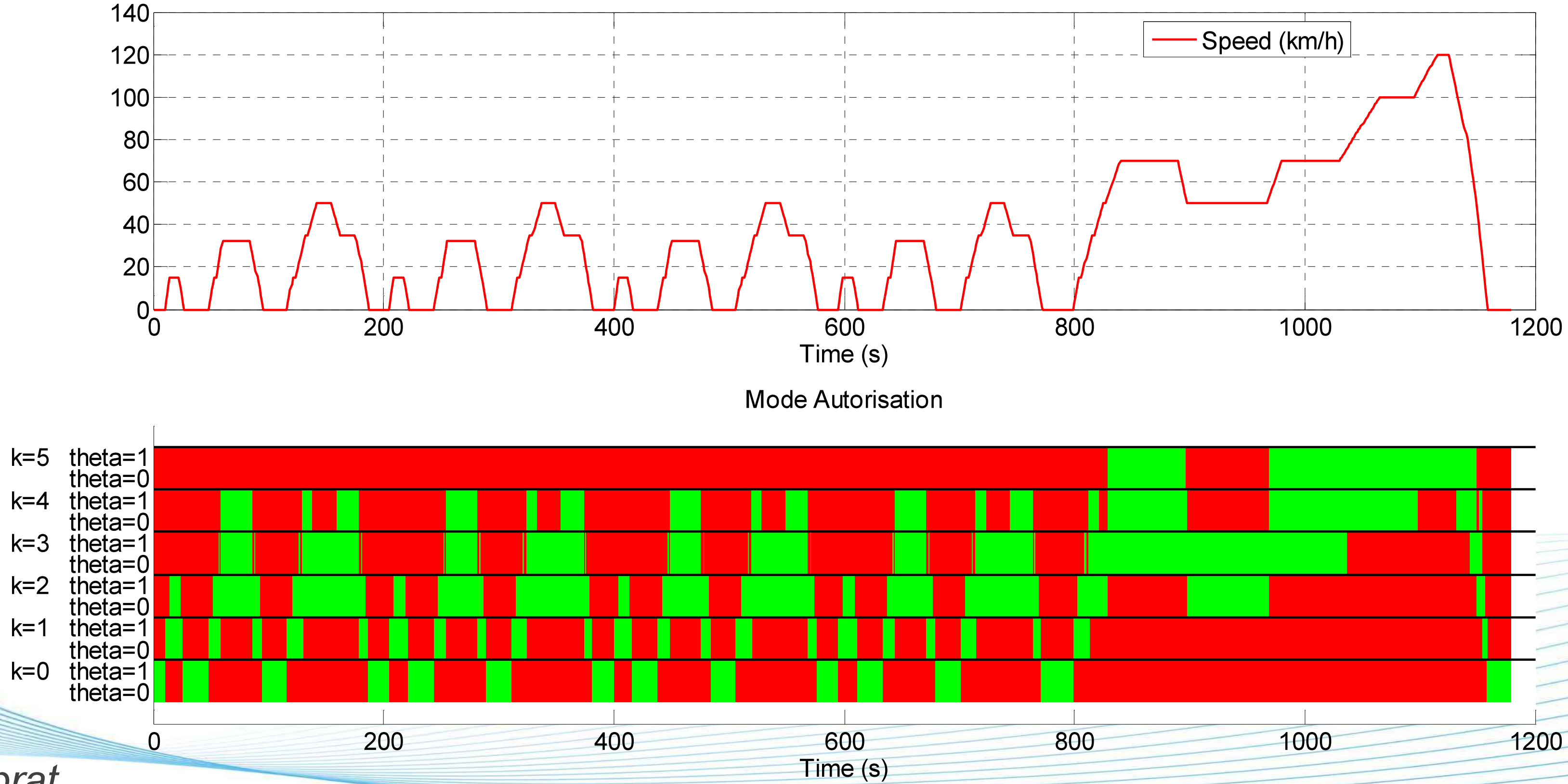


HEV management

Step 4 : Sets of possible values of the decisions variables

Continuous variables:

1 algebraic constraint : $T_w(t) = R\big(k(t)\big) \cdot (T_{ice}(t) + \rho \cdot T_{em})$

+ variables limitations: $T_{ice}(t) \in \left[\underline{T_{ice}}(t), \overline{T_{ice}}(t)\right]$ and $T_{em}(t) \in \left[\underline{T_{em}}(t), \overline{T_{em}}(t)\right]$

$\Rightarrow$ Interval algebra

Rewrite the constraint in the other ways:

- $\Leftrightarrow T_{ice}(t) = \frac{T_w(t)}{R(k(t))} - \rho \cdot T_{em}(t) \begin{cases} \overline{T_{ice}}(t) = \frac{T_w(t)}{R(k(t))} - \rho \cdot \underline{T_{em}}(t) \\ \underline{T_{ice}}(t) = \frac{T_w(t)}{R(k(t))} - \rho \cdot \overline{T_{em}}(t) \end{cases}$

- $T_{em} = \frac{1}{\rho}\frac{T_w(t)}{R(k(t))} - T_{ice}(t) \Leftrightarrow \begin{cases} \overline{T_{em}} = \frac{1}{\rho}\frac{T_w(t)}{R(k(t))} - \underline{T_{ice}}(t) \\ \underline{T_{em}} = \frac{1}{\rho}\frac{T_w(t)}{R(k(t))} - \overline{T_{ice}}(t) \end{cases}$

HEV management

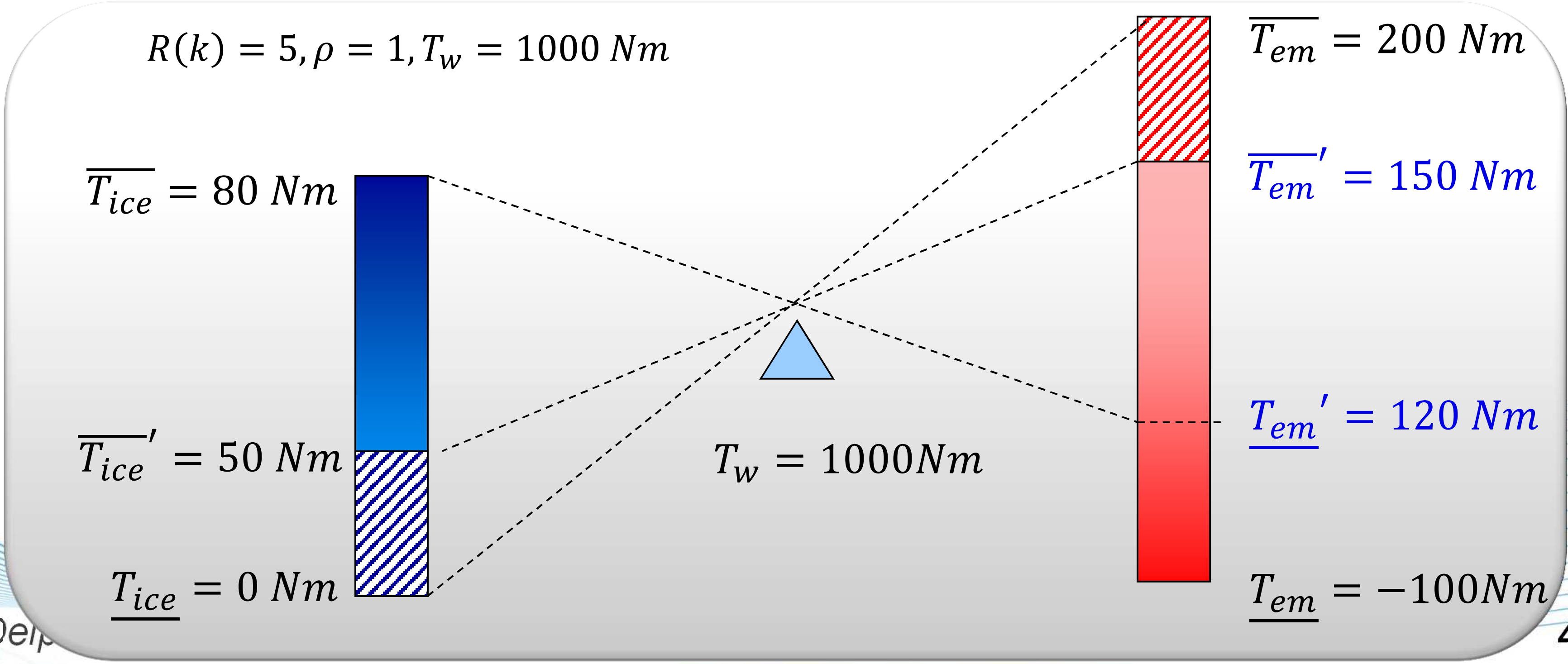
<u>Step 4 :</u> Sets of possible values of the decisions variables

<u>Continuous variables:</u>

1 algebraic constraint : $T_w(t) = R\big(k(t)\big) \cdot (T_{ice}(t) + \rho \cdot T_{em})$

+ variables limitations: $T_{ice}(t) \in \left[\underline{T_{ice}}(t), \overline{T_{ice}}(t)\right]$ and $T_{em}(t) \in \left[\underline{T_{em}}(t), \overline{T_{em}}(t)\right]$

$\Rightarrow$ Interval algebra



HEV management

<u>Step 4 :</u> Sets of possible values of the decisions variables

<u>Continuous variables:</u>

1 algebraic constraint : $T_w(t) = R\big(k(t)\big) \cdot (T_{ice}(t) + \rho \cdot T_{em})$

+ variables limitations: $T_{ice}(t) \in \left[\underline{T_{ice}}(t), \overline{T_{ice}}(t)\right]$ and $T_{em}(t) \in \left[\underline{T_{em}}(t), \overline{T_{em}}(t)\right]$

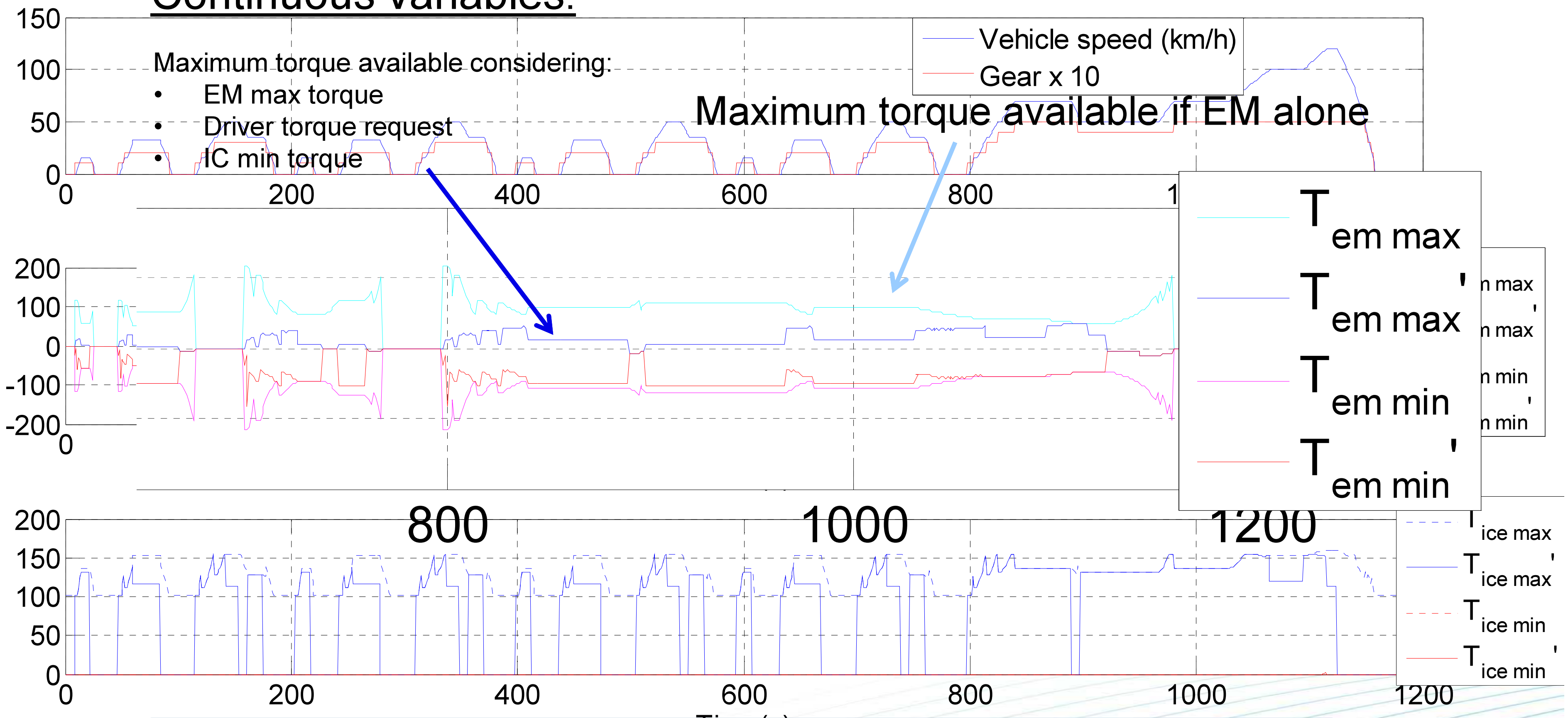
$\Rightarrow$ Interval algebra

- $$\left.\begin{array}{l} \overline{T_{ice}}'(t) = min\left(\overline{T_{ice}}(t), \frac{T_w(t)}{R(k(t))} - \rho \cdot \underline{T_{em}}(t)\right) \\ \underline{T_{ice}}'(t) = max\left(\underline{T_{ice}}(t), \frac{T_w(t)}{R(k(t))} - \rho \cdot \overline{T_{em}}(t)\right) \end{array}\right\} T_{ice}(t) \in \left[\underline{T_{ice}}'(t), \overline{T_{ice}}'(t)\right]$$

- $$\left.\begin{array}{l} \overline{T_{em}}'(t) = min\left(\underline{T_{em}}(t), \frac{1}{\rho}\frac{T_w(t)}{R(k(t))} - \underline{T_{ice}}(t)\right) \\ \underline{T_{em}}'(t) = max\left(\underline{T_{em}}(t), \frac{1}{\rho}\frac{T_w(t)}{R(k(t))} - \overline{T_{ice}}(t)\right) \end{array}\right\} T_{em}(t) \in \left[\underline{T_{em}}'(t), \overline{T_{em}}'(t)\right]$$

Step 4 : Sets of possible values of the decisions variables

Continuous variables:

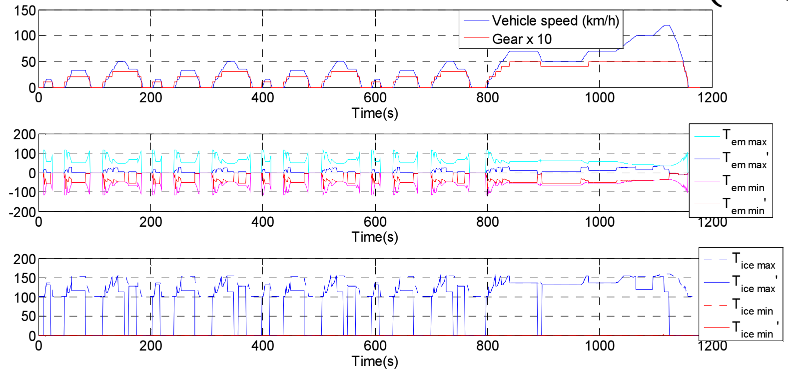


Finaly :

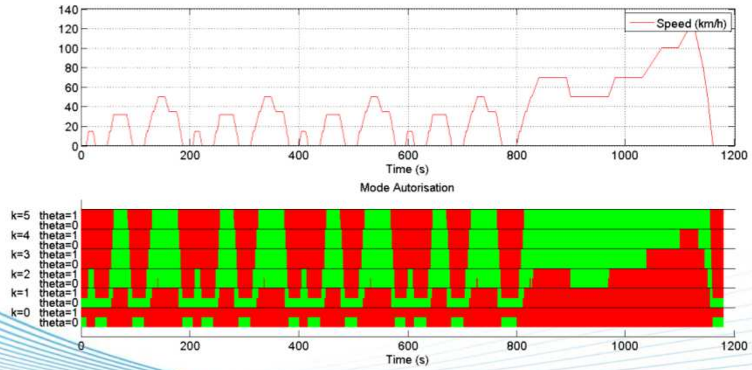
$$u_c(t) = \Phi_c(W(t)) \text{ with } \Phi_c(W(t)) = \left[\underline{T_{ice}}'(t), \overline{T_{ice}}'(t)\right]$$

Step 4 : Sets of possible values of the decisions variables

Continuous variables: $u_c(t) = \Phi_c\big(W(t)\big)$



Discrete variables: $u_d(t) = \Phi_d\big(W(t)\big)$



Set of possible control values

$$u(t) = [u_d(t) \quad u_c(t)] \in \Phi\big(W(t)\big)$$

With $\boldsymbol{\Phi}\big(\boldsymbol{W}(\boldsymbol{t})\big) = \boldsymbol{\Phi}_{\boldsymbol{d}}\big(\boldsymbol{W}(\boldsymbol{t})\big) \times \boldsymbol{\Phi}_{\boldsymbol{c}}\big(\boldsymbol{W}(\boldsymbol{t})\big)$

<u>Step 5 :</u> Rewriting the energy management as a classical optimal control problem

=> Algebraic constraints are removed

Fuel consumption: $J = \int_0^T \dot{m}_f\left(\omega_{ice}(t), T_{ice}(t)\right) \cdot \vartheta(t) \cdot dt$

Exogenous variables:

$$W(t) = \left[T_w(t) \quad \omega_w(t)\right]^T$$

Algebraic constraint :

$$\omega_{ice}(t) = \omega_w(t) \cdot R\left(k(t)\right)$$

$$Q\left(t, u(t), W(t)\right) = \dot{m}_f\left(\omega_w(t) \cdot R\left(k(t)\right), T_{ice}(t)\right)$$

Fuel consumption:

$$J = \int_0^T Q\left(t, u(t), W(t), x(t)\right) \cdot dt$$

Step 5 : Rewriting the energy management as a classical optimal control problem

=> Algebraic constraints are removed

System dynamics: $\dot{x}(t) = \frac{-1}{Q_{ah} \cdot 3600} I\left(\omega_{em}(t), T_{em}(t), x(t)\right)$

Exogenous variables:

$W(t) = \left[T_w(t) \quad \omega_w(t)\right]^T$

Algebraic constraints :

$\omega_{em}(t) = \omega_w(t) \cdot R(k(t)) \cdot \rho$

$T_{em}(t) = \frac{1}{\rho} \cdot \left(\frac{T_w(t)}{R(k(t))} - T_{ice}(t)\right)$

$$I_{ess}\left(t, u(t), W(t), x(t)\right) = I\left(\omega_w(t) \cdot R(k(t)) \cdot \rho, \frac{1}{\rho} \cdot \left(\frac{T_w(t)}{R(k(t))} - T_{ice}(t)\right), x(t)\right)$$

System dynamics:

$$\dot{x}(t) = f\left(t, u(t), W(t), x(t)\right)$$

HEV management

Step 5 : Rewriting the energy management as a classical optimal control problem

Fuel consumption:

$$J = \int_0^T \dot{m}_f\left(\omega_{ice}(t), T_{ice}(t)\right) \cdot \vartheta(t) \cdot dt$$

System dynamics:

$$\dot{x}(t) = \frac{-1}{Q_{ah} \cdot 3600} I\left(\omega_{em}(t), T_{em}(t), x(t)\right)$$

Constraints:

$$T_w(t) = R\left(k(t)\right) \cdot \left(T_{ice}(t) \cdot \vartheta(t) + \rho \cdot T_{em}(t)\right)$$

$$\omega_w(t) = \frac{\omega_{ice}(t)}{R\left(k(t)\right)} = \frac{\omega_{em}(t)}{R\left(k(t)\right) \cdot \rho}$$

$$T_{ice}(t) \in \left[\underline{T_{ice}}(t), \overline{T_{ice}}(t)\right] \qquad T_{em}(t) \in \left[\underline{T_{em}}(t), \overline{T_{em}}(t)\right]$$

$$x(t) \in \left[\underline{x}(t), \overline{x}(t)\right]$$

$$x(0) = x_0 \quad x(T) = x_T$$

$$J = \int_0^T Q\left(t, u(t), W(t), x(t)\right) \cdot dt$$

$$\dot{x}(t) = f\left(t, u(t), W(t), x(t)\right)$$

$$u(t) \in \Phi\left(W(t)\right)$$

$$\cancel{x(t) \in \left[\underline{x}(t), \overline{x}(t)\right]}$$

$$x(0) = x_0 \quad x(T) = x_T$$

Pontryagin Minimum Principle

<u>Theoretical limitations:</u>

- State constraints
  - Possible but quite complex /Herman 2009/
    Application to Hybrid Vehicle /Fontaine 2013/
  - Iterative algorithms: /Kulen & al. 2014/ /Rousseau 2008/
  - Penalty functions: /He & al. 2O12/ /Zhang & al. 2009

- Binary & integer variables
  - Theoretical work : /Riedinger 2003/
    Application to Hybrid Vehicle /Wei & al, 2007/ /Delprat & al. 2014/
  - Approximated solution : /Fan & Quin 2006/

<u>Modeling Hypothesis:</u>

- System dynamics and criterion are state in-dependent
  => to be discussed later

$$f\left(t,u(t),W(t),x(t)\right)\approx f\left(t,u(t),W(t),x_0\right) \qquad Q\left(t,u(t),W(t),x(t)\right)=Q\left(t,u(t),W(t),x_0\right)$$

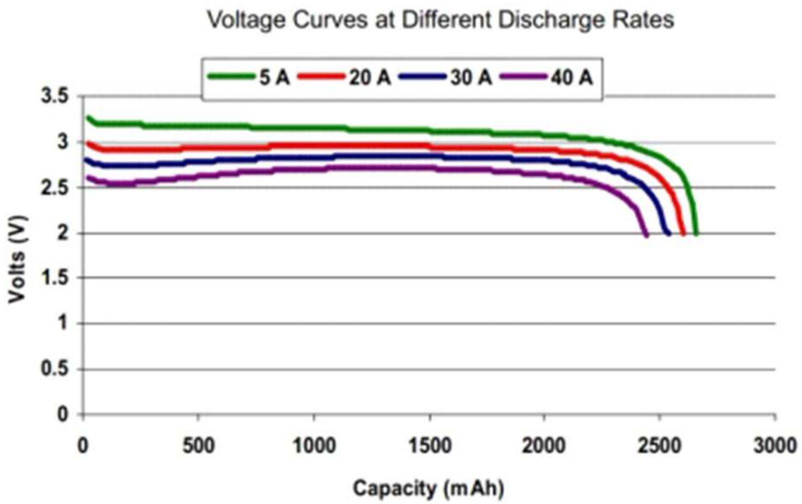
Pontryagin Minimum Principle

$f\big(t, W(t), x(t)\big) \approx f\big(t, W(t), x_0\big)$

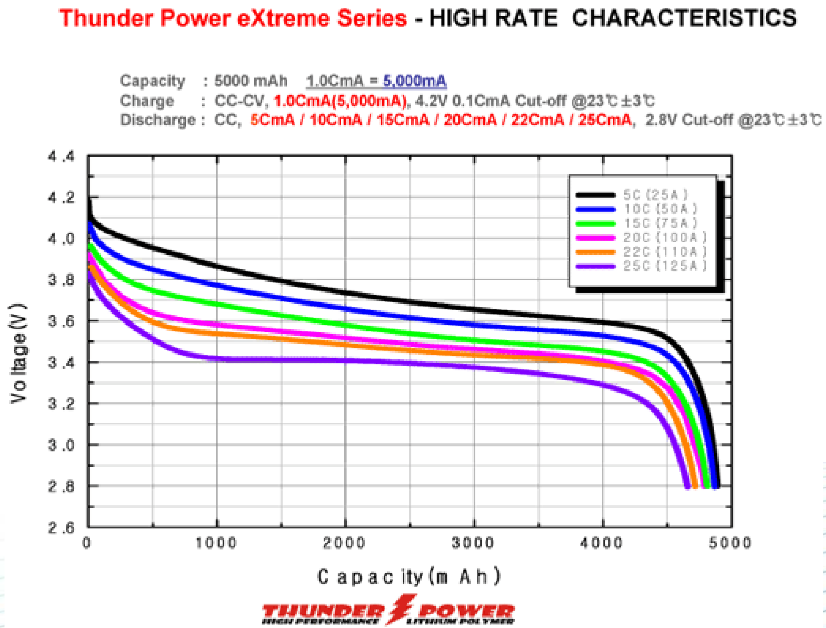
$\Leftrightarrow$ Open Circuit Voltage and Resistance does not depends on the Battery Soc

Two main points:

- *Charge sustaining control strategies*: design the control so the battery SOC remain close to a constant value
- Depending on the *battery technology*, the Voltage may be “quite” constant within the normal operating range



/LiFePO4 from K2 Energy/



/Thunder Power/

Pontryagin Minimum Principle

<u>Hypothesis and theoretical assumptions</u>

- There exists at least one optimal trajectory
- System and criterion are state independent
- Continuity & convexity assumptions

  Q and I_{ess} should be at least convex in u
- Engaged gear and IC engine state are fixed and known
- Driving cycle is known

Pontryagin Minimum Principle

VPPC 2014

- Hamiltonian /Pontryagin 1962/

$$H\left(t,u,x,W,\lambda\right)=\underbrace{Q\left(t,u,W,x_0\right)}_{\text{Fuel consumption}}+\overbrace{\lambda^T}^{\text{Co-state}}\cdot\underbrace{f\left(t,u,W,x\right)}_{\text{Electrical consumption}}$$

=> Physical interpretation

- Equivalent consumption
- λ represent the cost to convert electrical energy into fuel
- ECMS paradigm for real-time control

Pontryagin Minimum Principle

- <u>Hamiltonian</u>

$$H(t,u,x,W,\lambda) = \underbrace{Q(t,u,W,x_0)}_{\text{Fuel consumption}} + \overbrace{\lambda^T}^{\text{Co-state}} \cdot \underbrace{f(t,u,W,x)}_{\text{Electrical consumption}}$$

=> Optimality conditions along optimal trajectories (x^*, u^*, λ^*)

- $\dot{x}^*(t) = \left(\dfrac{\partial H}{\partial \lambda}\right)^*$ => System dynamics $\dot{x}^*(t) = f\left(t, u^*(t), W(t), x^*(t)\right)$
- $\dot{\lambda}^*(t) = -\dfrac{\partial H}{\partial x^*}\left(t, u^*(t), x^*(t), W(t), \lambda^*(t)\right)$ => Co-state dynamics $\dot{\lambda}(t) = 0$
- $H\left(t, u^*(t), x^*(t), W(t), \lambda^*(t)\right) \le H\left(t, u(t), x^*(t), W(t), \lambda^*(t)\right)$

Under (strict) convexity assumption

$$u^*(t) = \underset{v \in \Phi(W(t))}{\arg\min}\, H\left(t, v, x^*(t), W(t), \lambda^*(t)\right)$$

Pontryagin Minimum Principle

- <u>How to use these optimality conditions:</u>
  - Fix an (arbitrary) intial costate value $\lambda(0)$
  - For each time $t \in [0, T]$
    - Hamiltonian minimization $u^*(t) = \arg\min_{v \in \Phi(W(t))} H\left(t, v, x^*(t), W(t), \lambda^*(t)\right)$
    - Integrate the co-state dynamics: $\dot{\lambda}^*(t) = -\frac{\partial H}{\partial x^*}\left(t, u^*(t), x^*(t), W(t), \lambda^*(t)\right)$
    - Integrate the system dynamics: $\dot{x}^*(t) = f\left(t, u^*(t), W(t), x^*(t)\right)$
  - => The final state $x(T)$
  - If $x(T) \neq x_f$ then adapt $\lambda(0)$ (shooting algorithm)

Pontryagin Minimum Principle

VPPC 2014

Shooting algorithm: $\frac{\partial H}{\partial x} = 0$

Hamiltonian minimization : $\Pi\left(W,\lambda^*\right) = \underset{v\in\Phi(W)}{\arg\min}\, H\left(t,v,x^*(t),W\lambda^*\right)$

Final state : $x(t) = x_0 + \int_0^T f\left(t,\Pi\left(W(t),\lambda^*(t)\right),W(t),x^*(t)\right)\cdot dt$

$$g(\lambda_0) = x_0 + \int_0^T f\left(t,\Pi\left(W(t),\lambda_0^*\right),W(t),x^*(t)\right)\cdot dt$$

Need to solve : $g(\lambda_0) - x_f = 0$

=> Bisection method, Newton-based methods

Pontryagin Minimum Principle

- <u>Implementation consideration :</u>

  save computation at the expense of memory

  - Load driving cycle
  - Inverse dynamics => $T_w(i,k,\vartheta) \Rightarrow N \times N_{mode}$ matrix
  - ICE and EM limits : $\Rightarrow \bar{T}_{ice}(i,k,\vartheta) \Rightarrow N \times N_{mode}$ matrix

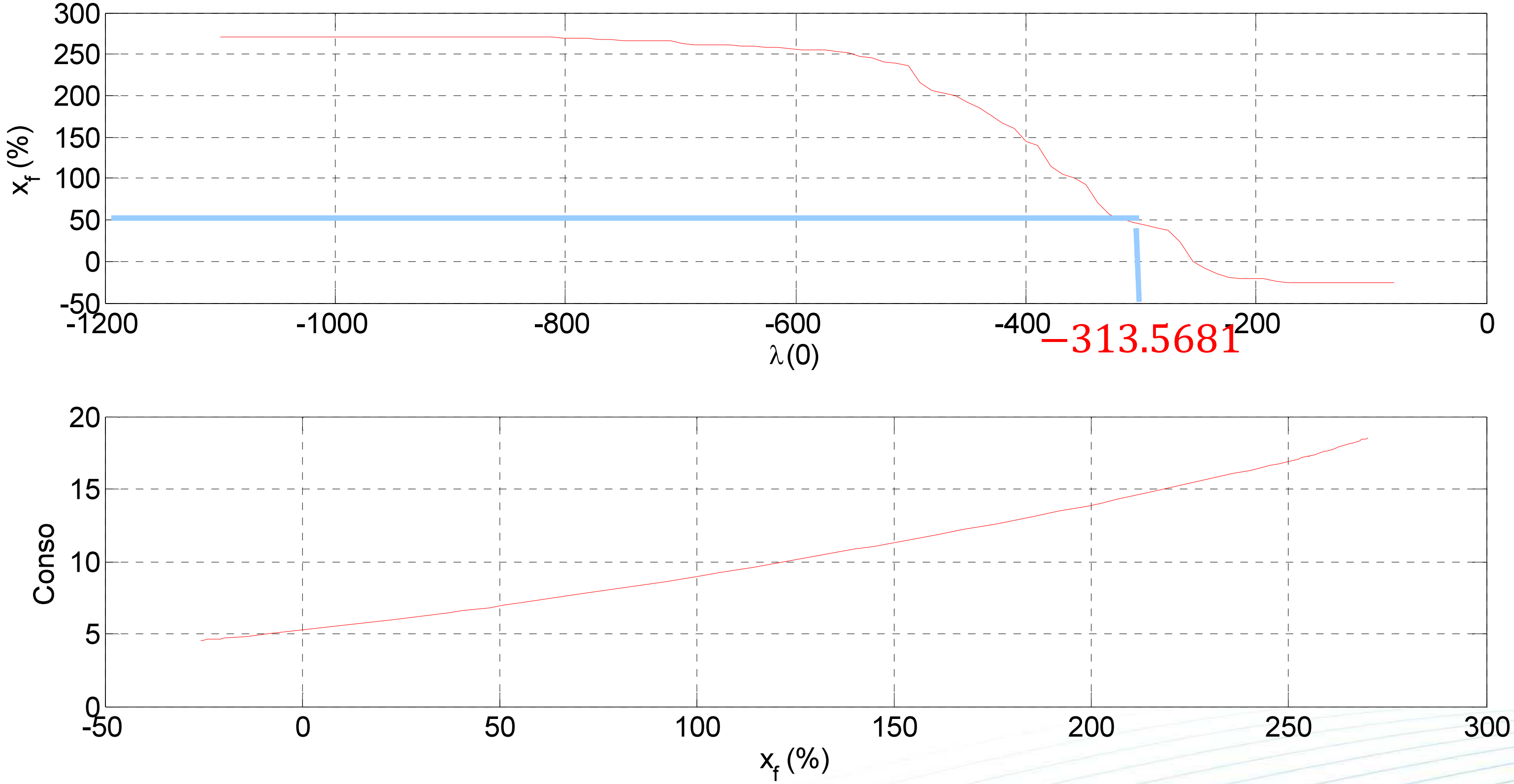
    $\Rightarrow \bar{T}_{em}(i,k,\vartheta) \Rightarrow N \times N_{mode}$ matrix
  - Effective control domain: $\bar{T}_{ice}(i,k,\vartheta), \underline{T}_{ice}(i,k,\vartheta) \Rightarrow N \times N_{mode}$ matrix
  - Sample enough values in the admissible interval and
    - Compute fuel at each sampling instants and control values
    - Compute current at each sampling instants and control values

=> All the complex calculus are pre-computed

$$H(t,u,x,W,\lambda) = \underbrace{Q(t,u,W,x_0)}_{\text{Pré-computed}} + \lambda^T \cdot \underbrace{f(t,u,W,x)}_{\text{Pré-computed}}$$

Pontryagin Minimum Principle

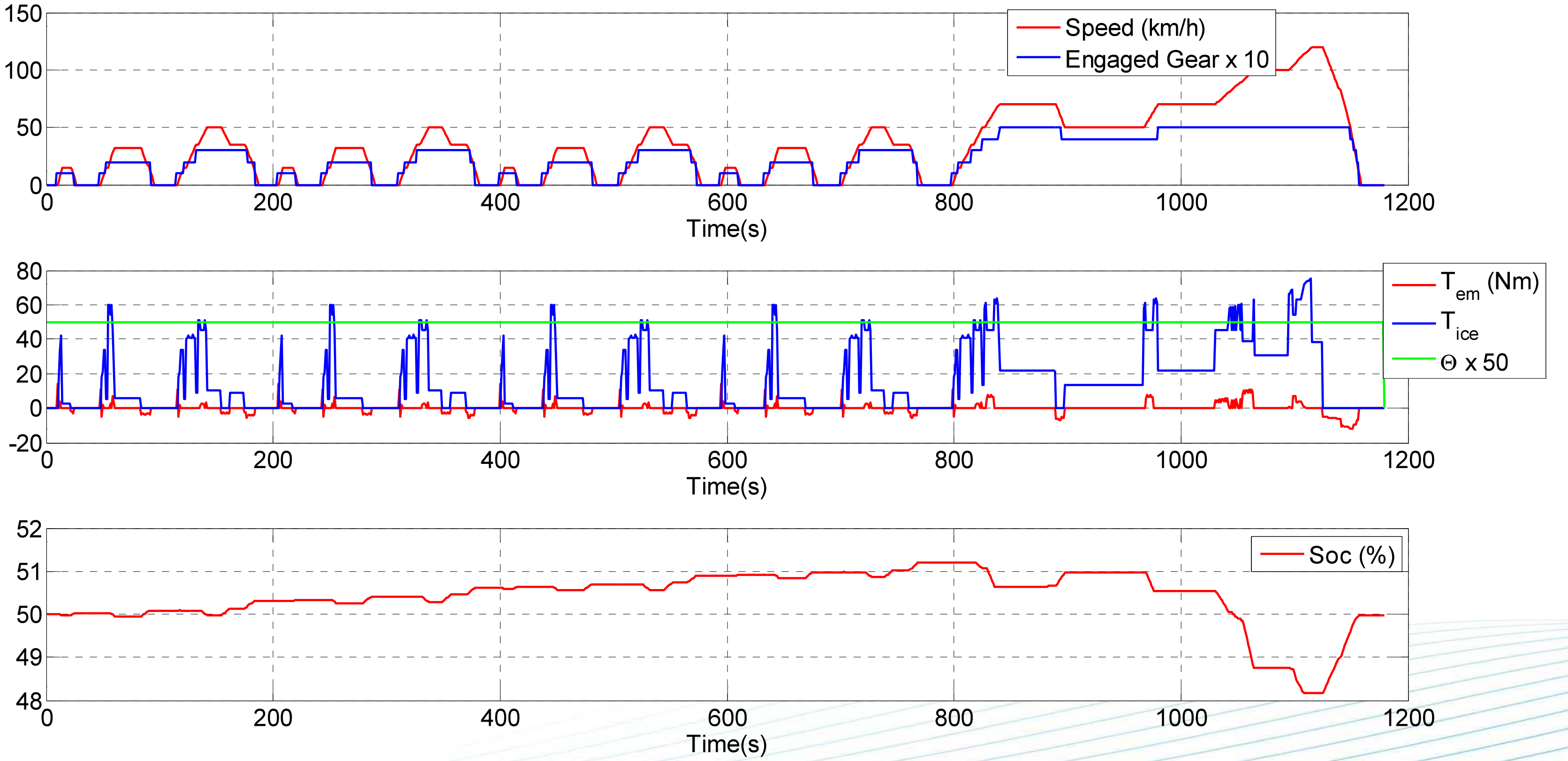
- Typical results :



Demo n°1

Pontryagin Minimum Principle

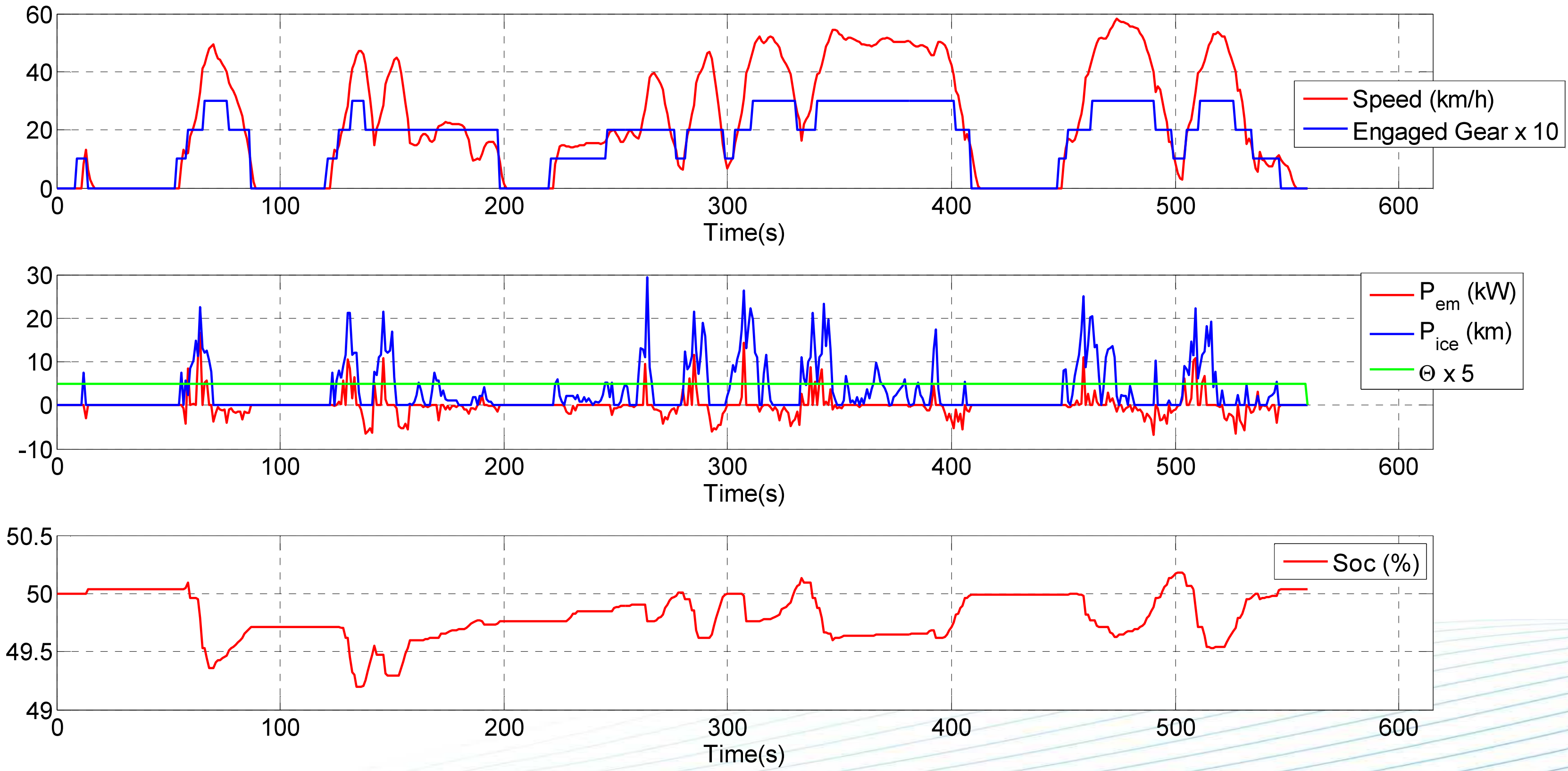
$x_0 = 50\%$ $x_f = 50\%$ $\lambda(0) = -313.5681$ => $x(T) = 50{,}02\%$
Fuel consumption : 6.85 l /100km (vs 7,06l/100km ⇔ 4,1% improvement)



Demo n°2

Pontryagin Minimum Principle

$x_0 = 50\%$ $x_f = 50\%$ $\lambda(0) = -327.15$=> $x(T) = 50{,}04\%$
Fuel consumption : 9,36 l /100km (vs 10,11 l/100km ⇔ 7,4% improvement)



Pontryagin Minimum Principle

Conclusion :

- Fuel consumption gains are quite poor ~4%
- The electric machine usage is low : 16kW vs 32kW available
- Batteries are useless : less than 1% usage

Engaged gear & IC engine state (on/off) must be optimized

Focus n°1 :

Mathematical models for :

- IC engine fuel consumption $Q(t, u, W, w)$
- Electric power consumption $f(t, u, W, w)$

Pontryagin Minimum Principle

VPPC 2014

Choosing a mathematical model for the fuel consumption & current

- *Polynomial approximation*

Assume there exist a polynomial model for fuel consumption and current:

$$Q(u,W) = a(W(t))\cdot u^2 + b(W(t))\cdot u + c(W(t))$$

$$f(u,W) = a'(W(t))\cdot u^2 + b'(W(t))\cdot u + c'(W(t))$$

<u>Convexity assumption :</u> $a(W(t)) > 0 \quad a'(W(t)) > 0$

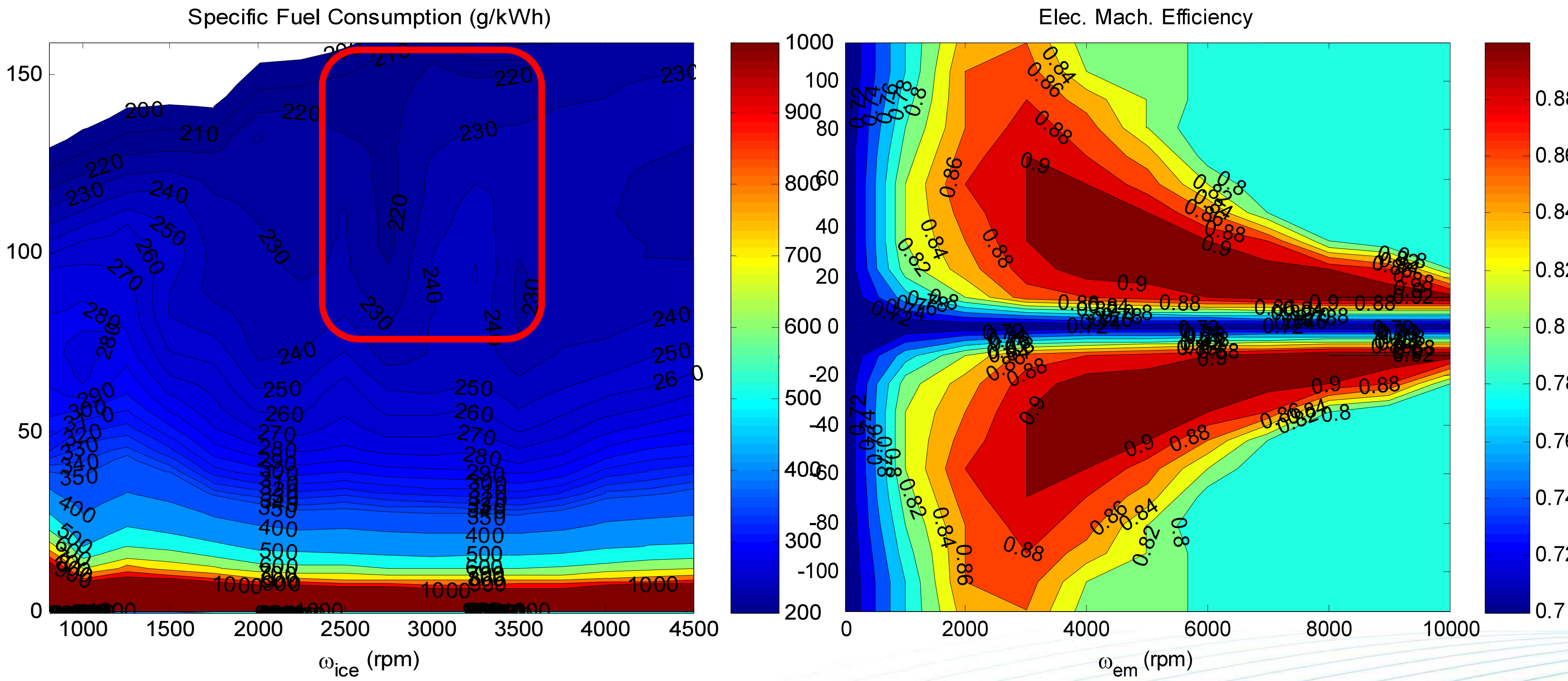
$$H(t,u,x,W,\lambda) = (a + \lambda^T \cdot a')\cdot u^2 + (b + \lambda^T \cdot b')\cdot u + c + \lambda^T \cdot c'$$

$$\arg\min H = \frac{-(b + \lambda^T \cdot b')^2 - \sqrt{\delta}}{2\cdot(a + \lambda^T \cdot a')} \qquad \delta = (b + \lambda^T \cdot b')^2 - 4\cdot(a + \lambda^T \cdot a')\cdot(c + \lambda^T \cdot c')$$

Pontryagin Minimum Principle

Choosing a mathematical model for the fuel consumption & current

- *Polynomial approximation*

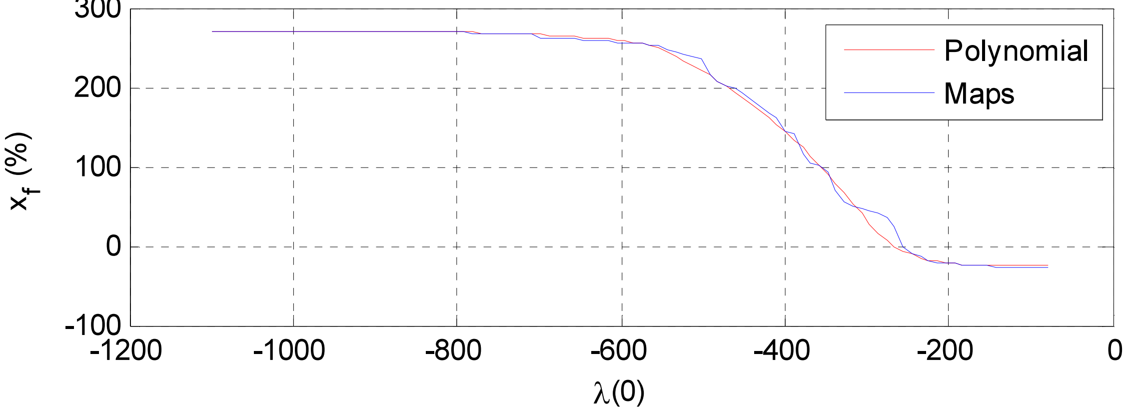


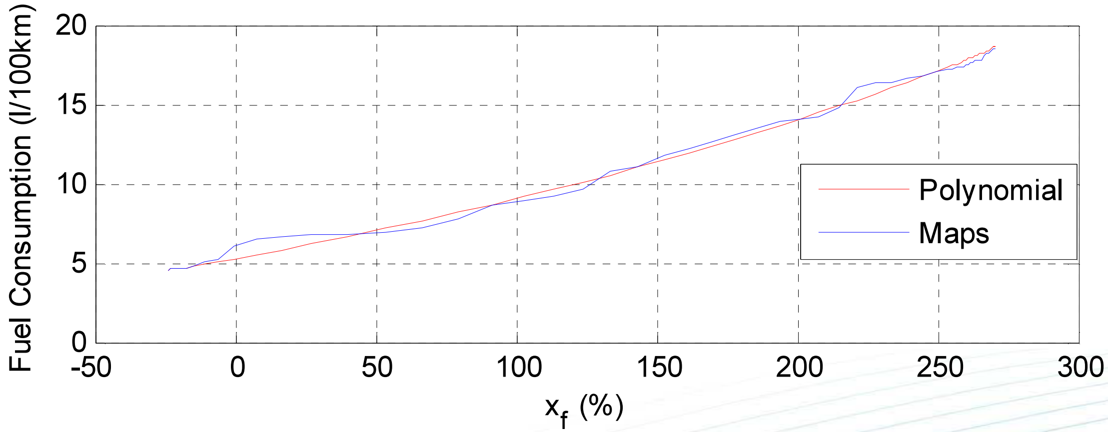
=> Actual engine data, with some “strange” behavior

Pontryagin Minimum Principle

Choosing a mathematical model for the fuel consumption & current

- *Polynomial approximation*





⇒ Fast and quite accurate approximation
⇒ Works better with “smooth” engine maps

Pontryagin Minimum Principle

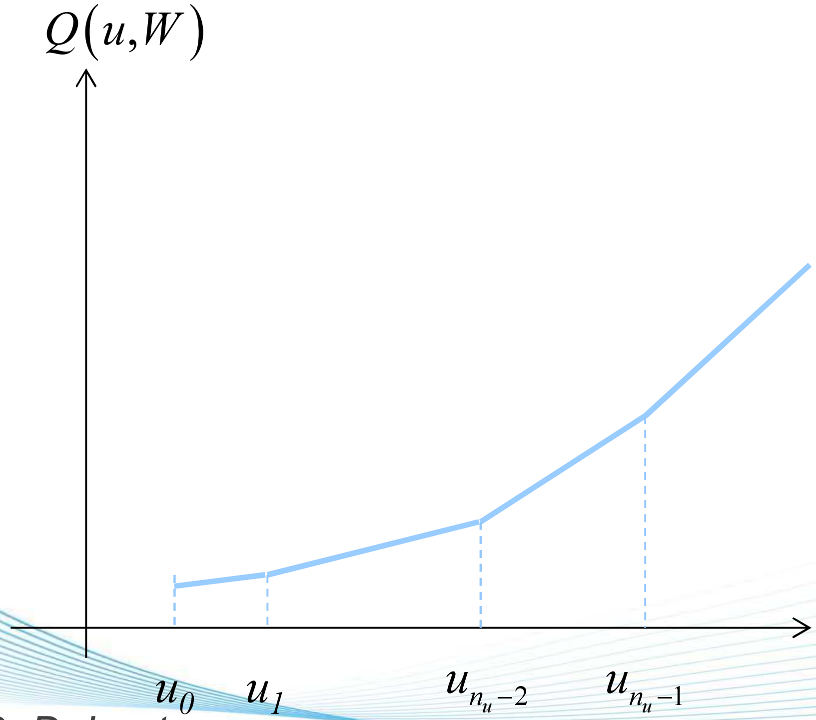
Choosing a mathematical model for the fuel consumption & current

- *Linear Interpolation over lookup table*

=> Piecewise linear models

$$Q(u,W) = a_i(W(t)) \cdot u + b_i(W(t)) \;\; \forall u \in [u_i, u_{i+1}] \quad i \in \{0,..,\mathrm{n}_u - 1\}$$

$$f(u,W) = c_i(W(t)) \cdot u + d_i(W(t)) \;\; \forall u \in [u_i, u_{i+1}] \quad i \in \{0,..,\mathrm{n}_u - 1\}$$



Convexity assumption:

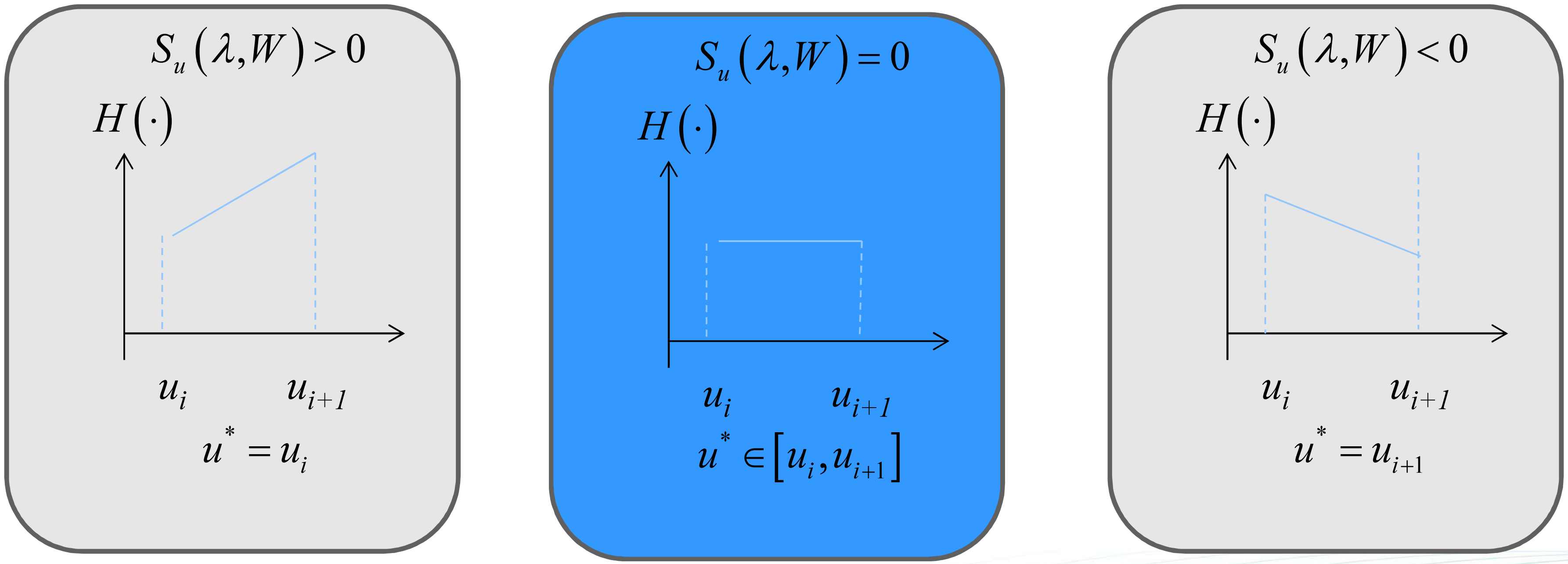
$$0 < a_0 < a_1 < \cdots < a_{n_u-2}$$

$$H(t,u,x,W,\lambda) = a_i \cdot u + b_i + \lambda^T \cdot (c_i \cdot u + d_i)$$

Choosing a mathematical model for the fuel consumption & current

- *Linear Interpolation over lookup table*

Hamiltonian is also piecewise linear $H(t,u,x,W,\lambda) = \left(\underbrace{a_i + \lambda^T \cdot c_i}_{S_i(\lambda,W)} \right) \cdot u + b_i + \lambda^T \cdot d_i$



Hamiltonian is not strictly convex, an infinite number of solutions can be constructed !! /Delprat & Hofman 2014/

Pontryagin Minimum Principle

The control is said to be singular: several control can be optimal if

$$S_u(\lambda, W) = 0 \Leftrightarrow \lambda = -\frac{a_i(W)}{c_i(W)}$$

For any $\lambda \neq -\frac{a_i(W)}{c_i(W)}$ the control is trivial : $u^* = u_i$ or $u^* = u_{i+1}$

For $\lambda = -\frac{a_i(W)}{c_i(W)}$ Hamiltonian minimization is a *multivalued* function:

$$u^* \in [u_i \, , u_{i+1}]$$

Remember, final state as a function of the co-state:

$$\Pi(W, \lambda^*) = \arg\min_{v \in \Phi(W)} H(t, v, x^*(t), W \lambda^*)$$

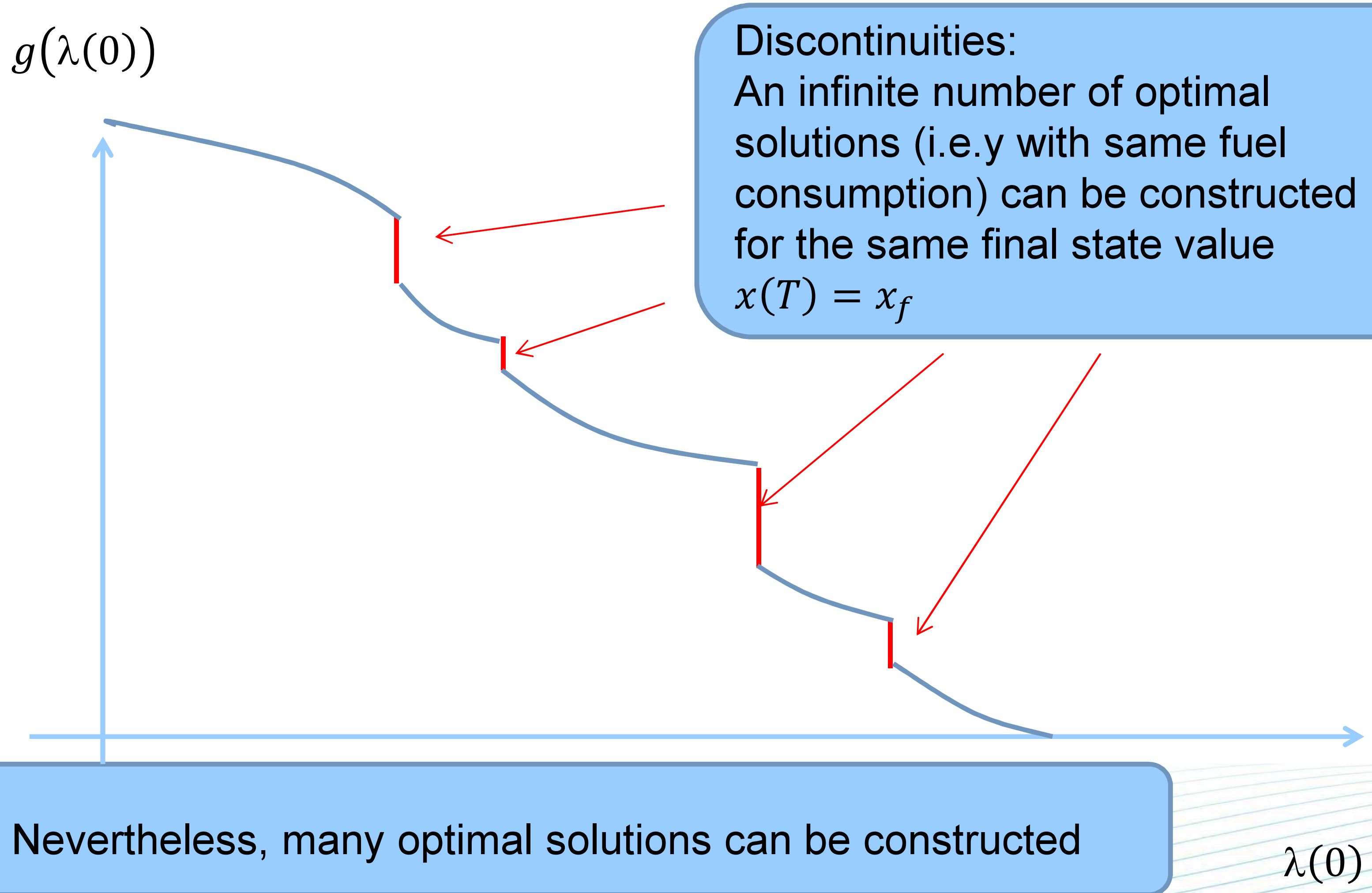
May be a scalar or a set

$$g(\lambda_0) = x_0 + \int_0^T f\left(t, \underbrace{\Pi(W(t), \lambda_0^*)}_{multi-valued \ \text{function}}, W(t), x^*(t)\right) \cdot dt$$

May also be a scalar or a set

Pontryagin Minimum Principle

The function $g(\lambda(0))$ is discontinuous

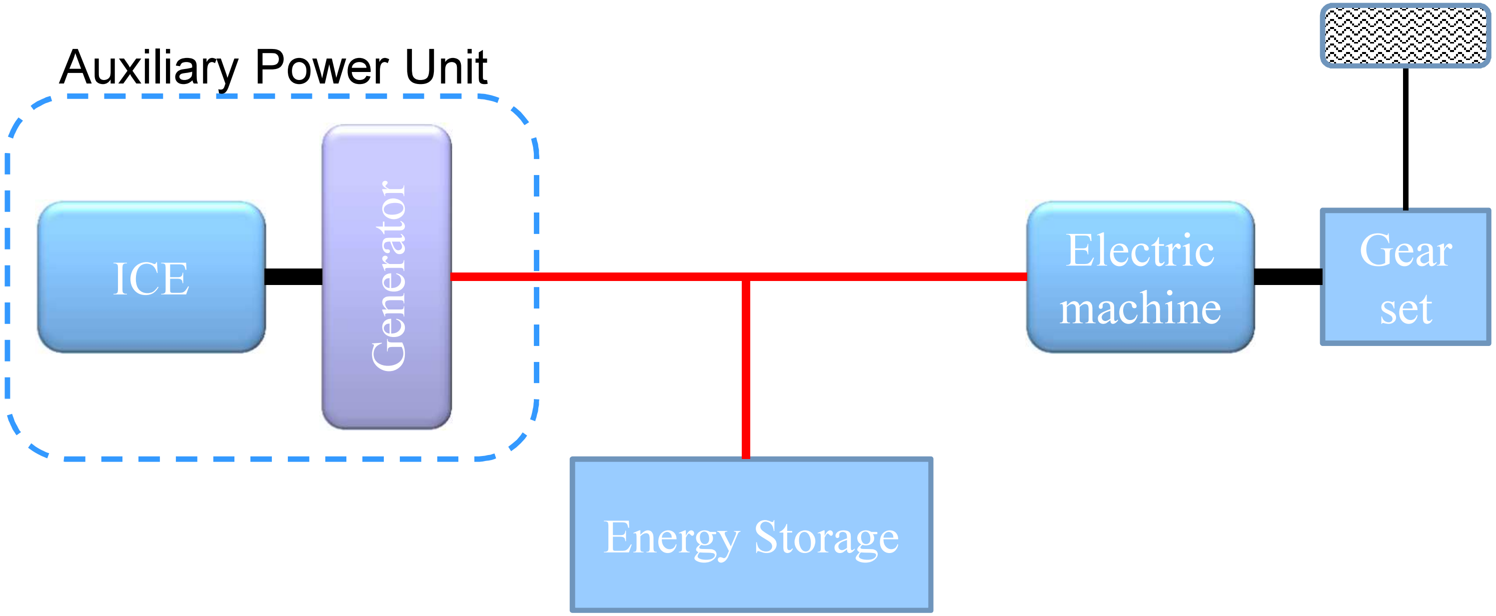


Discontinuities:
An infinite number of optimal solutions (i.e.y with same fuel consumption) can be constructed for the same final state value $x(T) = x_f$

Nevertheless, many optimal solutions can be constructed

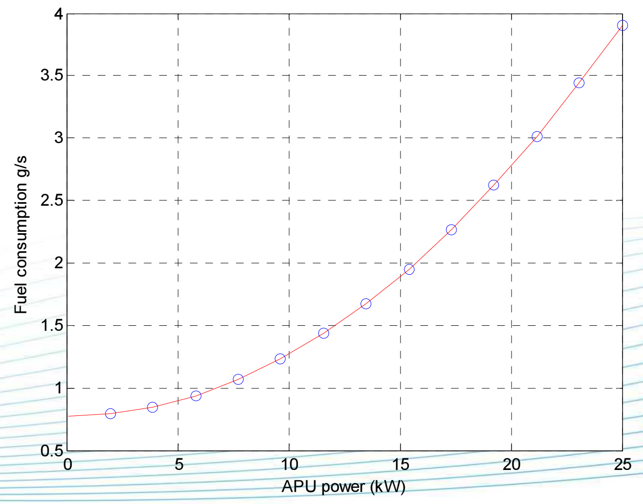
Pontryagin Minimum Principle

Example: Serial arrangement

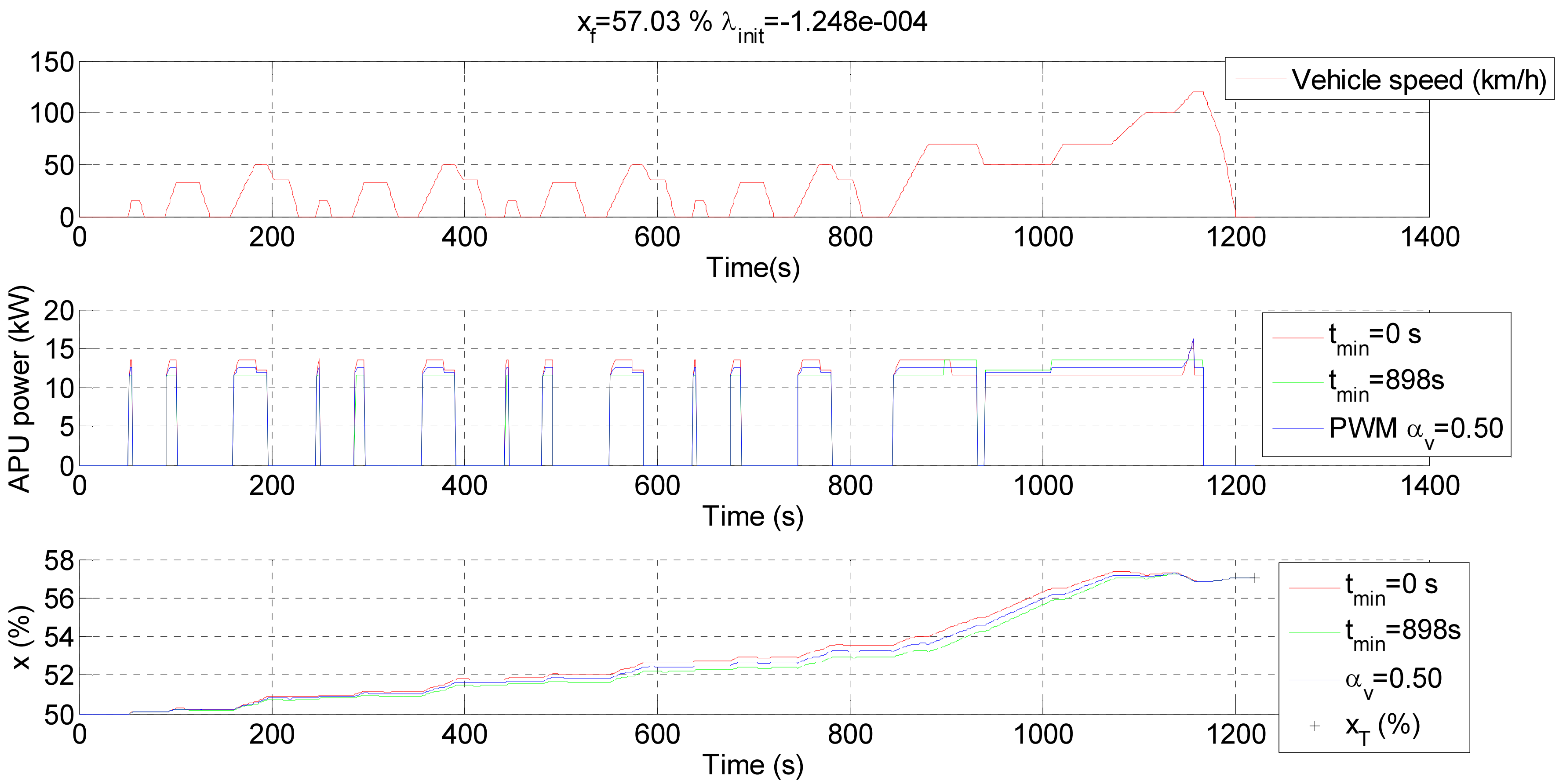


Auxiliary Power Unit : 25 kW
ESS max Power : 10 kW
ESS capacity : 40Ah, 300 V

Fuel consumption = f(APU power)
14 measurements, lookup table



Pontryagin Minimum Principle



All the solution have exactly the same fuel consumption : 2,68 l/100km

Demo n°1
Serial VPPC

Focus n°2 :

Binary and integer variable optimization :

- IC engine state on/off
- Fuel injection cut-off
- Engaged gear

Pontryagin Minimum Principle

Binary & integer variables optimization

=> Focus on binary variable optimization

$$u(t) = \left[u_c(t) \quad \vartheta(t) \right]^T$$

Pontryagin Minimum Principle requires continuous variables

Solution : Relaxation $\vartheta \in \{0,1\} \Rightarrow \vartheta \in [0,1]$ /Riedinger 2003/

Requires a careful analysis of optimal conditions

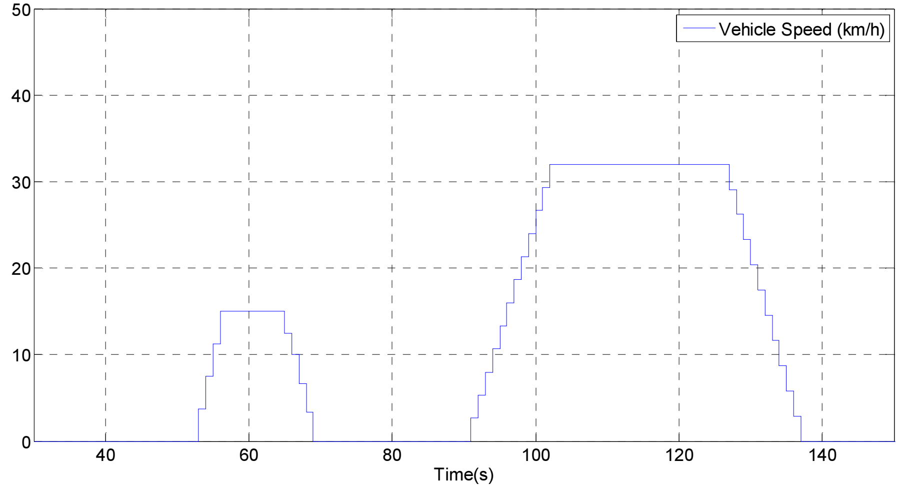
Implementations issues : drive cycle sampling

Pontryagin Minimum Principle

Binary & integer variables optimization

Driving cycle : piecewise constant data

$$v(t) = v_i \quad \forall t \in \left[i \cdot s, (i+1) \cdot s \right[$$



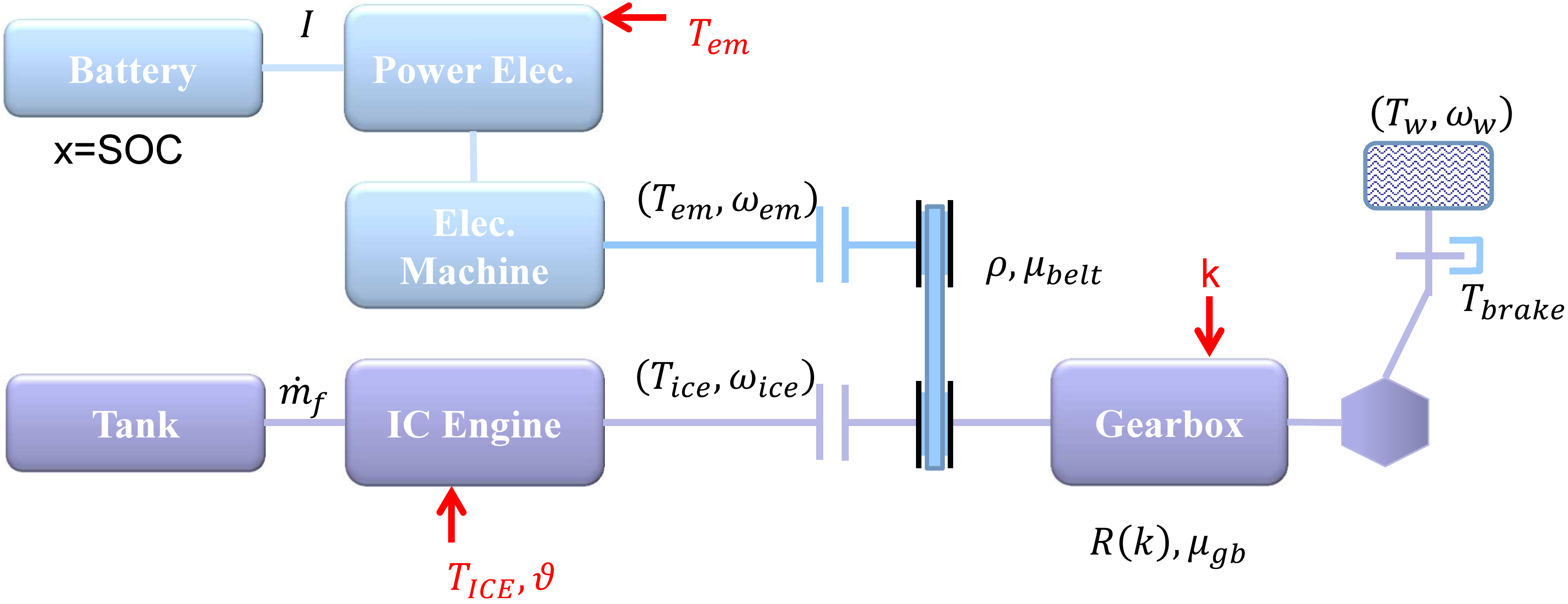
<u>Other solutions (linear interp)</u> requires advanced (Order 2 min) solver:
1) does not works well with binary variable switches (need adaptive steps)
2) Are not suitable for real-time extension

In many papers it is assumed that if the driving cycle is piecewise constant, then the control should also be piecewise constant… This assertion need to be checked…

$$u(t) \overset{???}{=} u_i \quad \forall t \in \left[i \cdot s, (i+1) \cdot s \right[$$

Pontryagin Minimum Principle

Case study: Parallel single shaft vehicle, Full Hybrid



Pontryagin Minimum Principle

Optimization problem

$$\vartheta(t) \in \{0,1\}$$

$$J = \int_0^T Q\left(t, \mathrm{u}_c(t), W(t), x(t)\right) \cdot \vartheta(t) \cdot dt$$

$$\dot{x}(t) = f\left(t, u(t), W(t), x(t)\right)$$

$$u(t) \in \Phi\left(W(t)\right)$$

$$x(0) = x_0 \quad x(T) = x_T$$

Suboptimal solution: Assume the previous PMP optimality conditions holds:

Hamiltonian:

$$H(t,u,x,W,\lambda) = \underbrace{Q\left(t, \mathrm{u}_c, W, x_0\right) \cdot \vartheta}_{\text{Fuel consumption}} + \overbrace{\lambda^T}^{\text{Co-state}} \cdot \underbrace{f(t,u,W,x)}_{\text{Electrical consumption}}$$

Optimal Control:

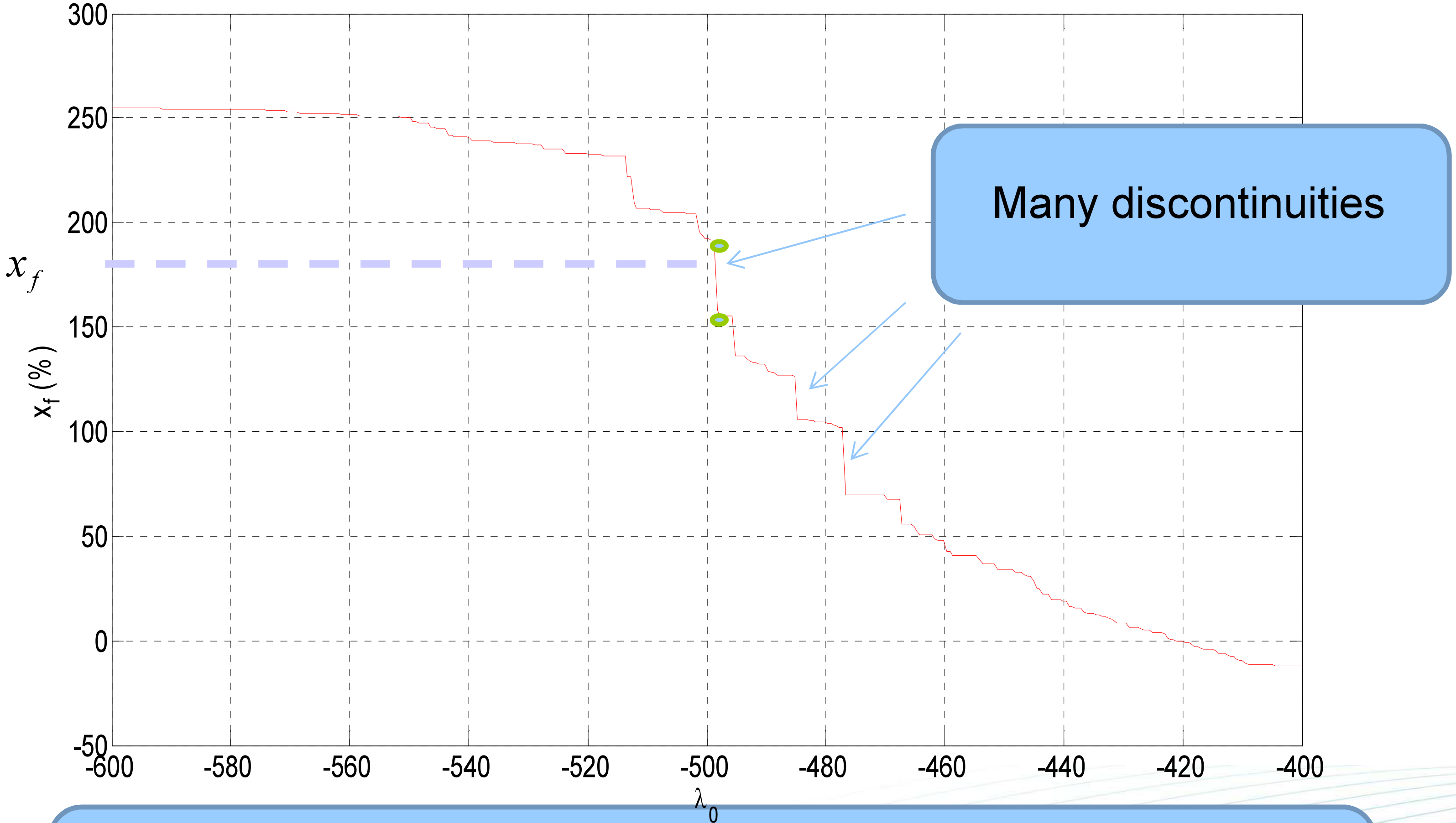
$$\begin{bmatrix} u_c & \vartheta \end{bmatrix}^T = \Pi\left(W, \lambda^*\right) = \underset{v \in \Phi(W)}{\arg\min}\, H\left(t, v, x^*(t), W \lambda^*\right)$$

Final state:

$$g(\lambda_0) = x_0 + \int_0^T f\left(t, \Pi\left(W(t), \lambda_0^*\right), W(t), x^*(t)\right) \cdot dt$$

Pontryagin Minimum Principle

Final state as a function of initial co-state:



Shooting algorithm will not work if x_f is on a discontinuity. The algorithm will converge … nuity extremum.

Demo n°3 Dichotic search

Pontryagin Minimum Principle

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Binary & integer variables optimization

Driving cycle : piecewise constant data

$$v(t)=v_i \quad \forall t\in\left[i\cdot s,(i+1)\cdot s\right[\quad \Leftrightarrow W(t)=W_i \quad \forall t\in\left[i\cdot s,(i+1)\cdot s\right[$$

Original Optimization Problem

$$\vartheta(t)\in\{0,1\}$$

$$J=\int_0^T Q\big(t,u(t),W(t),x(t)\big)\cdot\vartheta(t)\cdot dt$$

$$\dot{x}(t)=f\big(t,u(t),W(t),x(t)\big)$$

$$u(t)\in\Phi\big(W(t)\big)$$

$$x(0)=x_0 \quad x(T)=x_T$$

Relaxed optimization problem:

$$\vartheta(t)\in[0,1]$$

$$J=\int_0^T Q\big(t,u(t),W(t),x(t)\big)\cdot\vartheta(t)\cdot dt$$

$$\dot{x}(t)=f(\cdot)\cdot\vartheta(t)+\big(1-\vartheta(t)\big)\cdot f_0(\cdot)$$

$$u(t)\in\Phi\big(W(t)\big)$$

$$x(0)=x_0 \qquad x(T)=x_T$$

Convex relaxation

If the solutions of the Relaxed problem are bang-bang in ϑ,
then they are also solution for the original problem

Binary & integer variables optimization

Hamiltonian : $H(t,u,x,W,\lambda) = Q(t,u,W,x_0)\cdot\vartheta + \lambda^T\cdot\left[f(\cdot)\cdot\vartheta(t) + (1-\vartheta(t))\cdot f_0\right]$

1) <u>Continuous control u_c optimality condition</u> (does not depends on ϑ]) :

Let $\bar{u}_c$ be the Hamiltonian argmin (without saturation)

$$\frac{\partial H}{\partial \bar{u}_c} = 0 \Leftrightarrow \underbrace{\left(\frac{\partial Q(t,u,W,x_0)}{\partial \bar{u}_c} + \lambda^T\cdot\frac{\partial f(\cdot)}{\partial \bar{u}_c}\right)}_{u_c = \psi(t,x,W,\lambda)}\cdot\vartheta$$

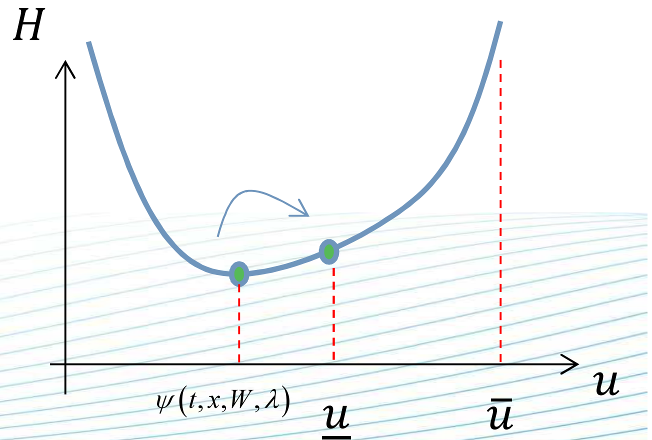
$$\vartheta = 0$$

H is assumed to be convex

<u>Assuming convex Hamiltonian:</u>

$$u_c = \psi_{sat}(t,x,W,\lambda) = sat\left(\psi(t,x,W,\lambda), \underline{u}(W), \bar{u}(W)\right)$$

=> The continuous control is fixed



Pontryagin Minimum Principle

Binary & integer variables optimization

Hamiltonian : $H\left(t,u,x,W,\lambda\right)=Q\left(t,u,W,x_0\right)\cdot\vartheta+\lambda^T\cdot\left[f\left(\cdot\right)\cdot\vartheta+\left(1-\vartheta\right)\cdot f_0\left(\cdot\right)\right]$

$$H\left(t,u,x,W,\lambda\right)=S_\vartheta\left(\cdot\right)\cdot\vartheta+\lambda^T\cdot f_0\left(\cdot\right) \qquad S_\vartheta\left(\cdot\right)=Q\left(t,\mathrm{u}_c,W,x_0\right)+\lambda^T\cdot\left(f\left(\cdot\right)-f_0\left(\cdot\right)\right)$$

As the continuous control is known ($u_c=\psi_{sat}(t,x,W,\lambda)$), S_ϑ can be 'simplified':

$$S_\vartheta\left(t,W,\lambda\right)=Q\left(t,\psi_{sat}\left(t,x,W,\lambda\right),W,x\right)+\lambda^T\cdot\left(f\left(\cdot\right)-f_0\left(\cdot\right)\right)$$

$$H\left(t,u,x,W,\lambda\right)=S_\vartheta\left(t,W,\lambda\right)\cdot\vartheta+\lambda^T\cdot f_0\left(\cdot\right)$$

2) The binary variable is fixed according to $sign(S_\vartheta(\cdot))$:

- $\mathrm{S}_\vartheta(\cdot)>0 \Leftrightarrow u(t)=\vartheta(t)$=0
- $\mathrm{S}_\vartheta(\cdot)<0 \Leftrightarrow u(t)=\vartheta(t)$=1
- What if $\mathrm{S}_\vartheta(\cdot)=0$?
  => Singular binary control, $\vartheta(t)\in[0,1]$

Pontryagin Minimum Principle

Binary & integer variables optimization

Let us note, Φ the set of co-state values leading to singular binary variable: $S_\vartheta(t, \lambda, w_i) = 0 \ \forall \lambda \in \Phi$:

$$S_\vartheta(t,W,\lambda) = Q\left(t,\psi_{sat}(t,x,W,\lambda),W,x\right) + \lambda^T \cdot \left(f(\cdot) - f_0(\cdot)\right)$$

Taking into account the saturation can be complex.

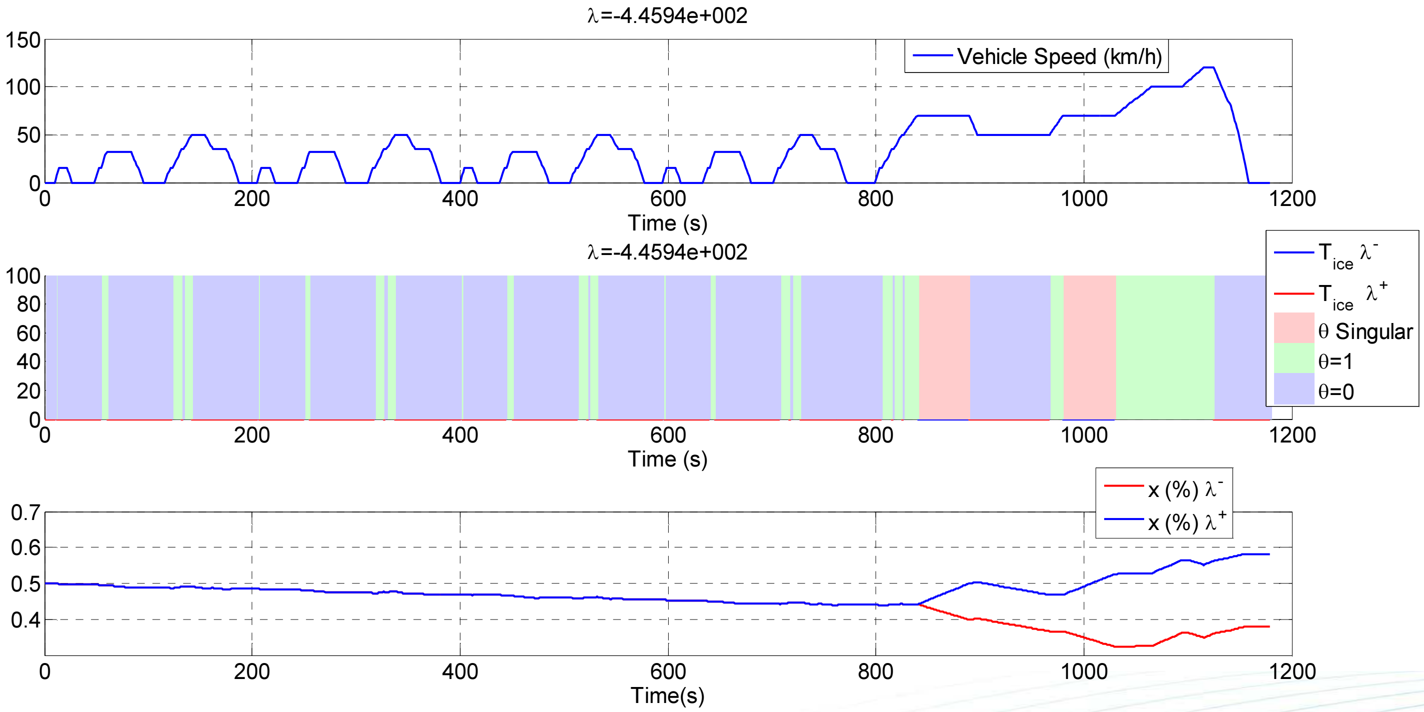
$$\Phi = \left\{ \lambda = -\frac{Q\left(t,\psi_{sat}(t,x,w_i,\lambda),w_i,x\right)}{f(t,x,w_i,\lambda) - f_0(\cdot)}, i = 0..n_w - 1 \right\}$$

Pontryagin Minimum Principle

Binary & integer variables optimization

Remark : if $\lambda \in \Phi$, then :

$S_\vartheta(t,\lambda,w_i))=0$ for some w_i $\qquad$ $S_\vartheta(t,\lambda,w_i))\neq 0$ for some w_i



$$I_{nonDef} = \{840..890, 980..1030\}$$

$$I_{wellDef} = \textit{the other samples index}$$

Pontryagin Minimum Principle

Binary & integer variables optimization

Driving cycle is piecewise constant: $S_\vartheta(t,\lambda,w_i))$ is also piecewise constant

Let us define the set of time t with well defined control:

- $I_{wellDefOn} = \{i = 0..n_w - 2 : S_\vartheta(t,\lambda,w_i) < 0\}$
- $I_{wellDefOff} = \{i = 0..n_w - 2 : S_\vartheta(t,\lambda,w_i) > 0\}$
- $I_{nonDef} = \{i = 0..n_w - 2 : S_\vartheta(t,\lambda,w_i) = 0\}$

<u>Activation function:</u>

$$\Gamma(\mathrm{I}_x, t) = \begin{cases} 1 & if\ \ t \in \bigcup_{i \in I_x} \left[i \cdot s, (i+1) \cdot s \right[\\ 0 & otherwise \end{cases}$$

Now : all the possible <u>optimal</u> state trajectories can be defined

Pontryagin Minimum Principle

Binary & integer variables optimization

<u>Final state:</u> $\dot{x}(t) = f(\cdot)\cdot\vartheta(t) + (1-\vartheta(t))\cdot f_0(\cdot)$

$$x(T) = x_0 + \int_0^T \left[\left(\underbrace{f\left(t,\psi,P_{on}(t,x,W,\lambda),W(t),d(t)\right)}_{P_{on}(t,x,W,\lambda)} - f_0(\cdot)\right)\cdot\vartheta(t) + f_0(\cdot)\right]\cdot dt$$

$$\boxed{\int_0^T f_0(\cdot)\,dt}$$

$\int_0^T P_{on}(t,x,W,\lambda)\cdot 1\cdot\Gamma\left(\mathrm{I}_{wellDefOn},t\right)\cdot dt$ Fixed amount of energy for $\vartheta(t)=1$

$\int_0^T P_{on}(t,x,W,\lambda)\cdot 0\cdot\Gamma\left(\mathrm{I}_{wellDefOff},t\right)\cdot dt$ Fixed amount of energy for $\vartheta(t)=0$

$\boxed{\int_0^T P_{on}(t,x,W,\lambda)\cdot\vartheta(t)\cdot\Gamma\left(\mathrm{I}_{nonDef},t\right)\cdot dt}$ => Fixed amount of energy to ensure x(T)=xf with $\vartheta \in [0,1]$

$$x(T) = x_0 + \boxed{\int_0^T P_{on}(t,x,W,\lambda)\cdot\vartheta(t)\cdot\Gamma\left(\mathrm{I}_{nonDef},t\right)\cdot dt} + \boxed{h(\lambda)}$$

Constant and known

Pontryagin Minimum Principle

Binary & integer variables optimization

$$x(T) = x_0 + \int_0^T P_{on}(t,x,W,\lambda)\cdot \vartheta(t)\cdot \Gamma\left(\mathrm{I}_{nonDef},t\right)\cdot dt + h(\lambda)$$

Piecewise constant driving cycle: $W(t) = w_i \; i = 0..n_w - 1$

$$h(\lambda) = \sum_{\mathrm{I}_{wellDefOn}} P_{on}(t,x,W,\lambda)\cdot s + \sum_{i=0}^{n_w-1} f_0(\cdot)\cdot s$$

Final state constraint :$x(T) = x_f$

So all the optimal state trajectories must satisfy

$$\int_0^T P_{on}(t,x,W,\lambda)\cdot \vartheta(t)\cdot \Gamma\left(\mathrm{I}_{nonDef},t\right)\cdot dt = x_f - x_0 - h(\lambda)$$

Amongst all of them, could we construct a few ones with $\boldsymbol{\vartheta(t) \in [0,1]}$?

Pontryagin Minimum Principle

Binary & integer variables optimization

<u>Constraint</u> $$\underbrace{\int_0^T P_{on}(t,x,W,\lambda)\cdot \vartheta(t)\cdot \Gamma\left(\mathrm{I}_{nonDef},t\right)\cdot dt}_{\text{Amount of energy to be produced when } \vartheta(t)=1} = x_f - x_0 - h(\lambda)$$

<u>Simplest solution :</u>

- $\vartheta(t) = 1$ until the amount of energy is produced,
- $\vartheta(t) = 0$ after

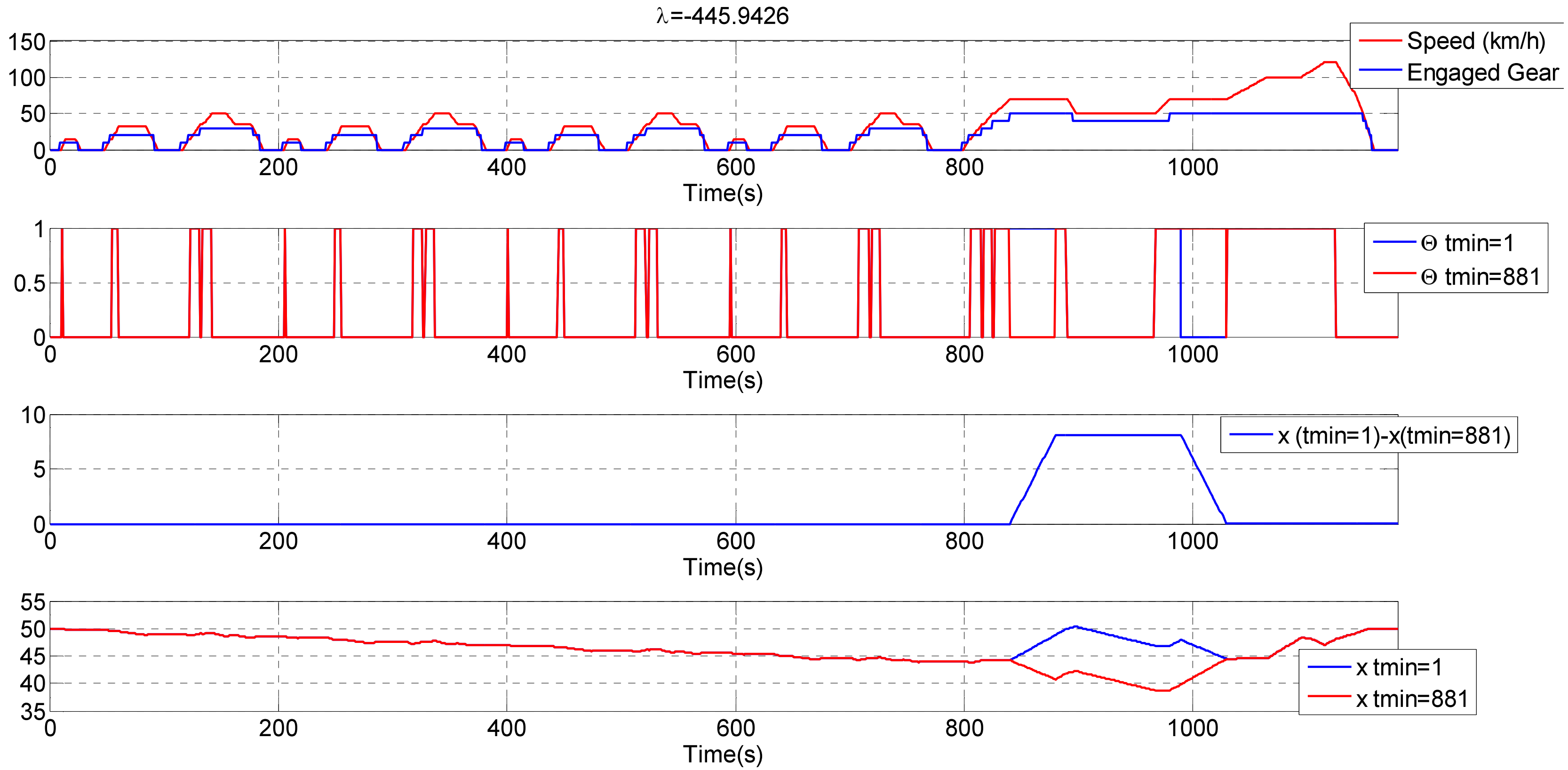
In other words: Fix t_{min} and find t_{max} such that:

$$\int_{t_{\min}}^{t_{\max}} P_{on}(t,x,W,\lambda)\cdot 1\cdot \Gamma\left(\mathrm{I}_{nonDef},t\right)\cdot dt = x_f - x_0 - h(\lambda)$$

Pontryagin Minimum Principle

Binary & integer variables optimization

$x_f = 50\% \quad \lambda = -445{,}9426$



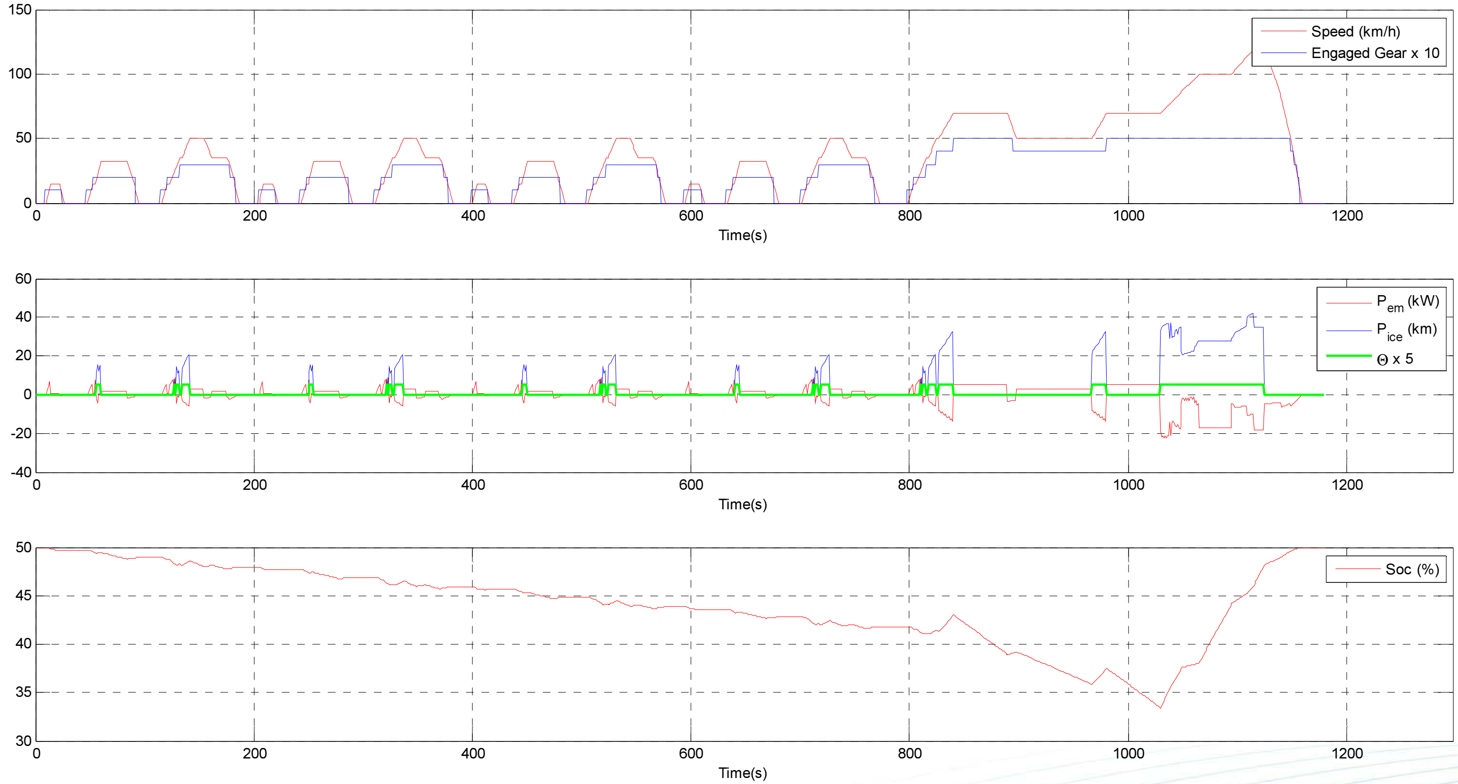
Fuel consumlption : 3.95 l/100 km/h for both trajectories

$x(T) - x_f = -2.7756 \cdot \mathbf{10^{-14}}\ \%$

Pontryagin Minimum Principle

NEDC Optimization results (Gear + IC engine optimized)

Fuel consumption = 3,75 l/100km $x_f = 50\%$



Pontryagin Minimum Principle

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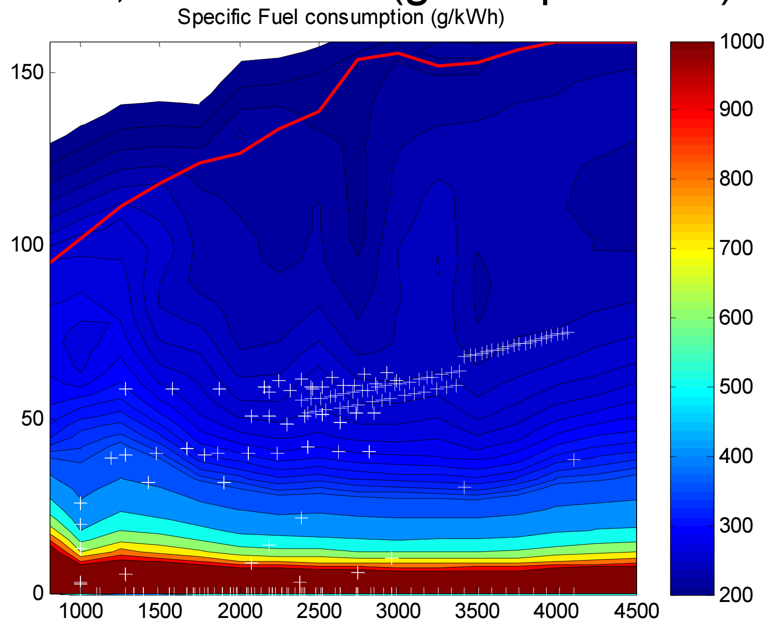
NEDC Optimization results (Gear + IC engine optimized)

$x_f = 50\%$

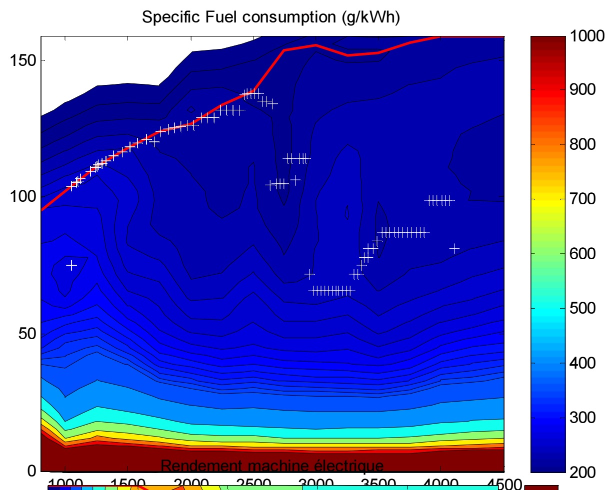
Conventional vehicle :

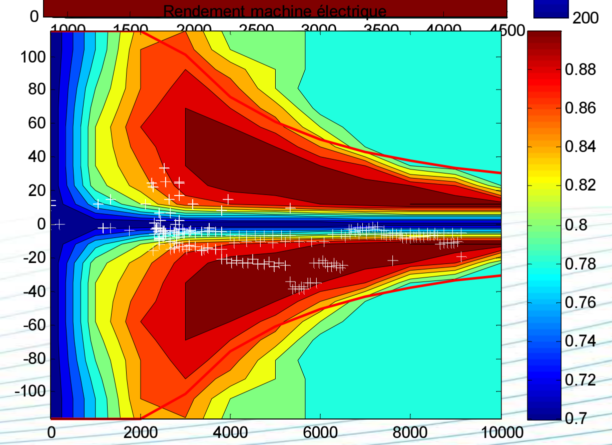
- 7,06 l/100 km (gear fixed)
- 6,77 l/100 km (gear optimized)



Hybrid vehicle:

- 3,75 l/100km (gear + ICE State)



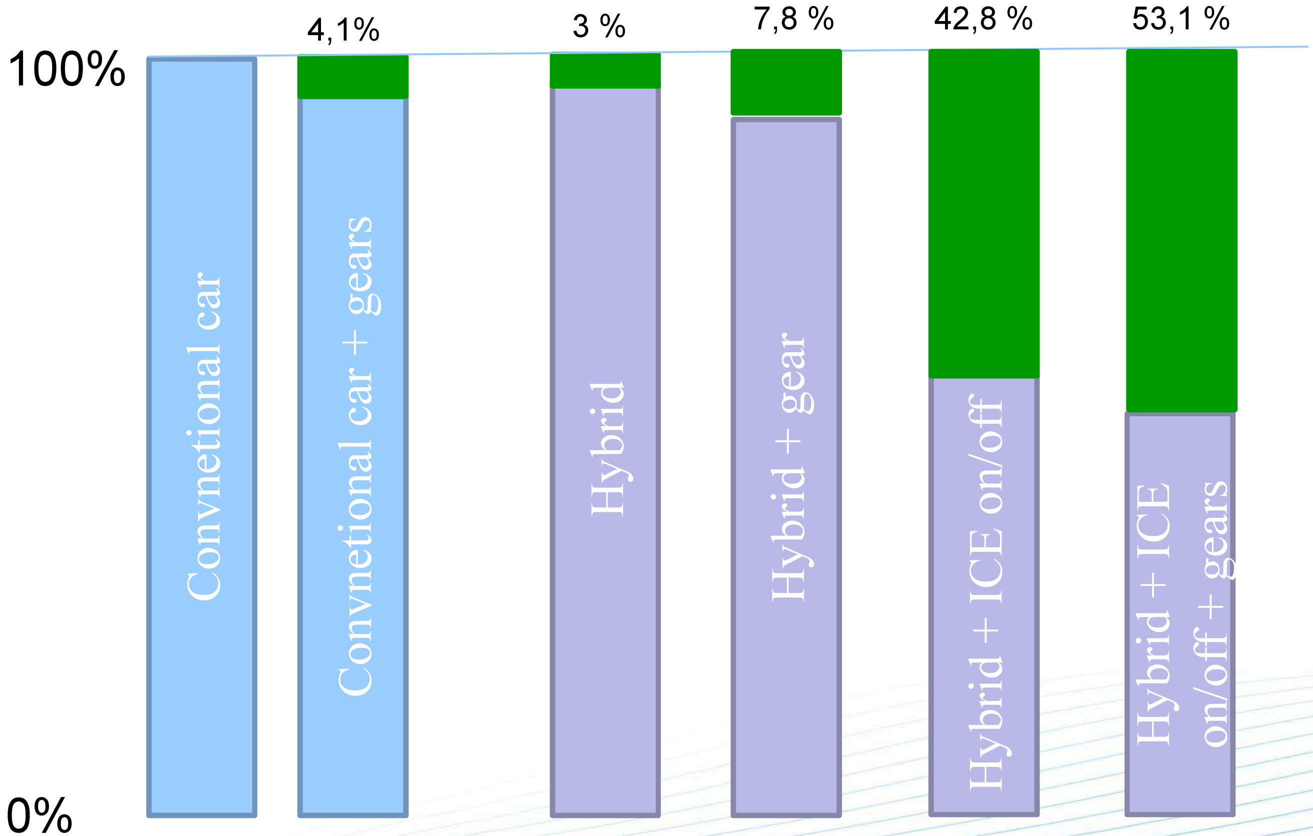


Pontryagin Minimum Principle

NEDC Optimization results (Gear + IC engine optimized)

$x_f = 50\%$

Most interesting feature : hybrid + ICE on/off



Focus n°3 :

State constraints:

- Penalty function
- Iterative algorithms

$$J = \int_0^T Q\left(t, u(t), W(t), x(t)\right) \cdot dt$$

$$\dot{x}(t) = f\left(t, u(t), W(t), x(t)\right)$$

$$u(t) \in \Phi\left(W(t)\right)$$

State constraint → $x(t) \in \left[\underline{x}(t), \overline{x}(t)\right]$

$$x(0) = x_0$$

$$x(T) = x_T$$

Pontryagin Minimum Principle

State Constraints

Most of the Energy Storage System have technological limits:

<u>Supercapacitors :</u>

- $Voltage \in [0, V_{max}]$
- Power Electronics often requires $Voltage > V_{min}$

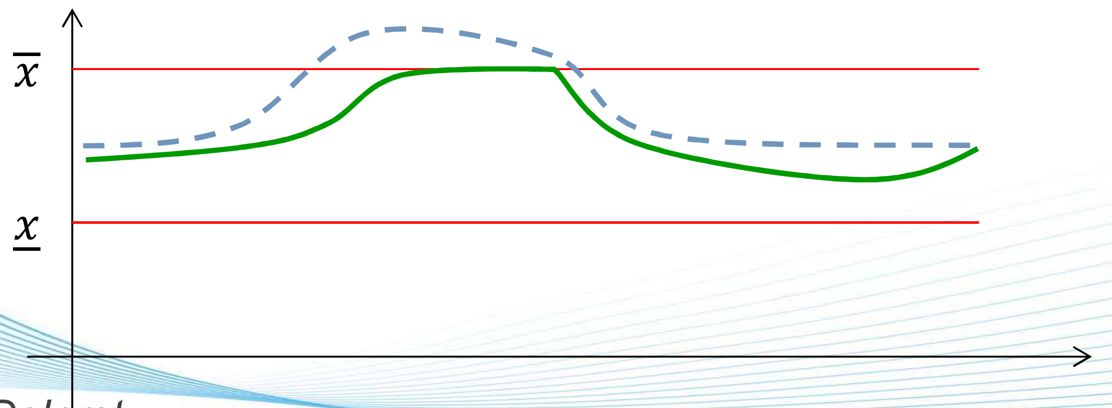
$\Rightarrow Voltage \in [V_{min}, V_{max}]$

$\Rightarrow Energy \in \left[\frac{1}{2} C \cdot {V_{min}}^2, \frac{1}{2} C \cdot {V_{max}}^2\right]$

<u>Li-ion batteries:</u>

=> $Voltage \in [V_{min}, V_{max}]$

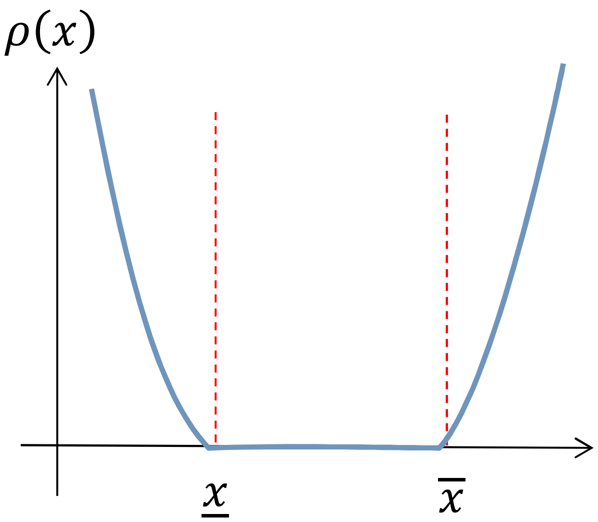
=> Soc$\in [SOC_{min}, SOC_{max}]$



Pontryagin Minimum Principle

State Constraints : Penalty functions

Simplest approach: increase the fuel cost when the state exceed limits



Penalty function:

$$\rho(x)=\begin{cases}(x-\underline{x})^2 & if \quad x<x\\ 0 & if \quad x\in\left[\underline{x},\overline{x}\right]\\ (\overline{x}-x)^2 & if \quad x>\overline{x}\end{cases}$$

Modified **optimization problem:**

$$J=\int_0^T Q\left(t,u(t),W(t),x(t)\right)+\alpha\cdot\rho(x)\cdot dt$$

$$\dot{x}(t)=f\left(t,u(t),W(t),x(t)\right)$$

$$u(t)\in\Phi\left(W(t)\right)$$

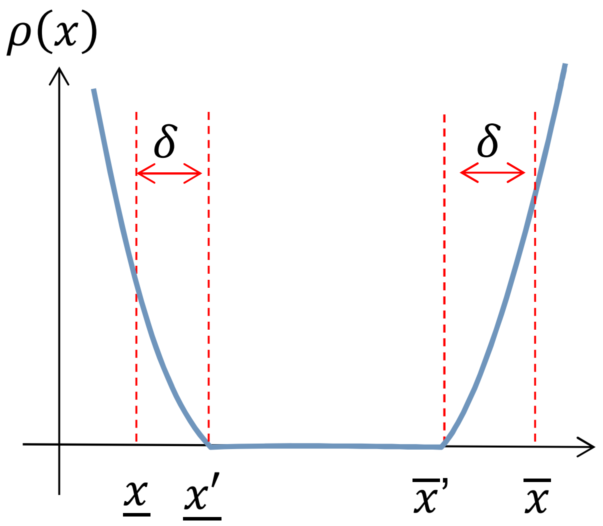
$$x(0)=x_0$$

$$x(T)=x_T$$

Pontryagin Minimum Principle

State Constraints : Penalty functions

Simplest approach: increase the fuel cost when the state exceed limits



<u>Penalty function:</u>

$$\rho(x)=\begin{cases}(x-\underline{x}+\delta)^2 & if\ \ x<x\\ 0 & if\ \ x\in[\underline{x},\overline{x}]\\ (\overline{x}+\delta-x)^2 & if\ \ x>\overline{x}\end{cases}$$

<u>*Modified* optimization problem:</u>

$$J=\int_0^T Q\big(t,u(t),W(t),x(t)\big)+\alpha\cdot\rho(x)\cdot dt$$

$$\dot{x}(t)=f\big(t,u(t),W(t),x(t)\big)$$

$$u(t)\in\Phi\big(W(t)\big)$$

$$x(0)=x_0$$

$$x(T)=x_T$$

Pontryagin Minimum Principle

State Constraints : Penalty functions

Optimality conditions

$$H(t,u,x,W,\lambda) = \underbrace{\underbrace{Q(t,u,W,x)}_{\text{Fuel consumption}} + \underbrace{\alpha \cdot \rho(x)}_{\textit{Penalty}}}_{\text{Modified fuel consumption}} + \overbrace{\lambda^T}^{\text{Co-state}} \cdot \underbrace{f(t,u,W,x)}_{\text{Electrical consumption}}$$

$$u^*(t) = \arg\min_{v \in \Phi(W(t))} H\left(t,v,x^*(t),W(t),\lambda^*(t)\right)$$

$$\dot{\lambda}^*(t) = -\frac{\partial H}{\partial x^*} = -\underbrace{\frac{\partial Q}{\partial x}}_{=0} - \alpha \cdot \frac{\partial \rho}{\partial x} - \lambda^T \cdot \underbrace{\frac{\partial f}{\partial x}}_{=0}$$

$$\dot{\lambda}(t) = -\alpha \cdot \frac{\partial \rho}{\partial x}$$

Considering the co-state as an equivalence factor between the fuel and the electric power, the cost increase when state reaches the limits

Pontryagin Minimum Principle

State Constraints : Additional dynamics

/D. Kirk 2004/

- Idea : Consider a new state which final value should be reach a given value if and only if the solution comply with the state constraint

Additionnal dynamics :

$$\frac{dy}{dt} = \rho(x) \geq 0$$

$$\rho(x) = \begin{cases} (x-\underline{x})^2 & if \quad x < x \\ 0 & if \quad x \in [\underline{x}, \overline{x}] \\ (\overline{x}-x)^2 & if \quad x > \overline{x} \end{cases}$$

Associated trajectory:

$$y(t) = y_0 + \int_0^T \rho(x(t)) \cdot dt$$

$$y(T) = y_0 \Leftrightarrow x(t) \in [\underline{x}, \overline{x}] \quad \forall t \in [0, T]$$

So $y(t)$ can only reach a prescribed final value $y_f = y_0$ if the state trajectory is admissible with regard to the constraint.

Same Hamiltonian expression as before

$$H(t,u,x,W,\lambda) = \underbrace{\underbrace{Q(t,u,W,x)}_{\text{Fuel consumption}} + \underbrace{\alpha \cdot \rho(x)}_{Penalty}}_{\text{Modified fuel consumption}} + \overbrace{\lambda^T}^{\text{Co-state}} \cdot \underbrace{f(t,u,W,x)}_{\text{Electrical consumption}}$$

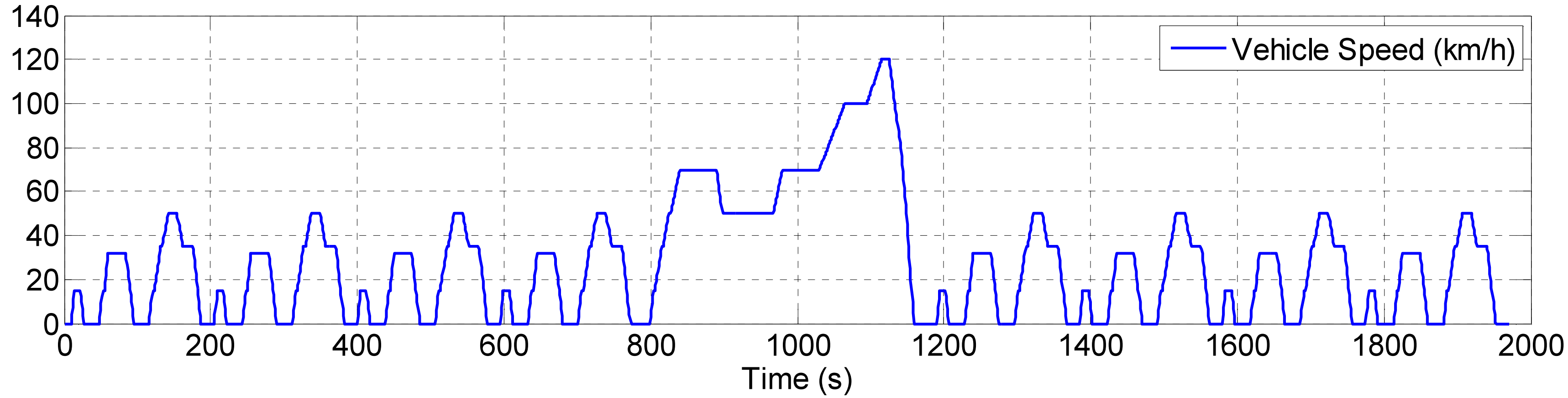
Pontryagin Minimum Principle

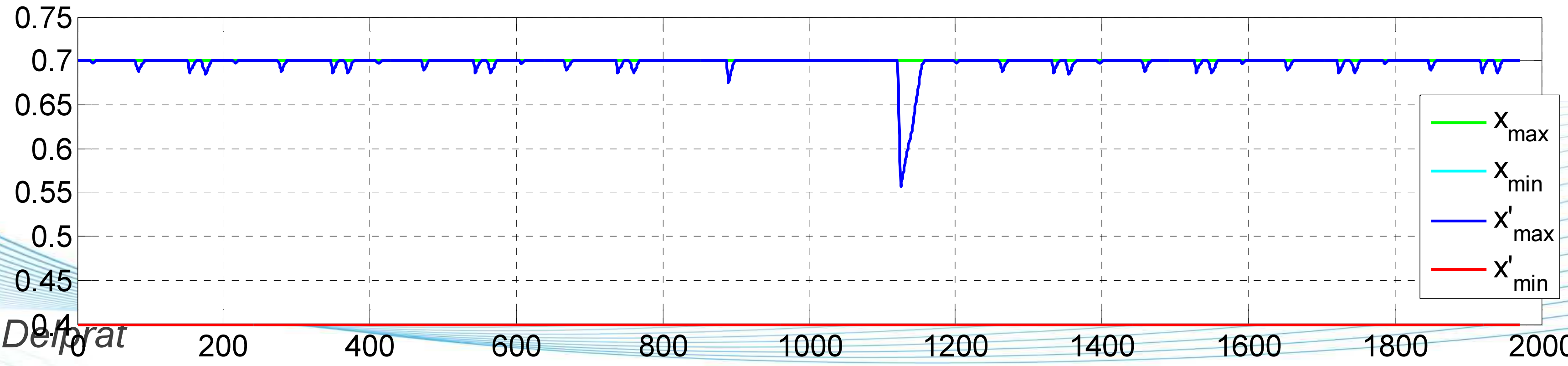
State Constraints : Additional dynamics /D. Kirk 2004/

Many problems:

- Does an optimal solution exists with the state constraint?

  $\Rightarrow$ Need a careful analysis of the optimization problem & energetics models
- Need to compute the max&min feasible state trajectory





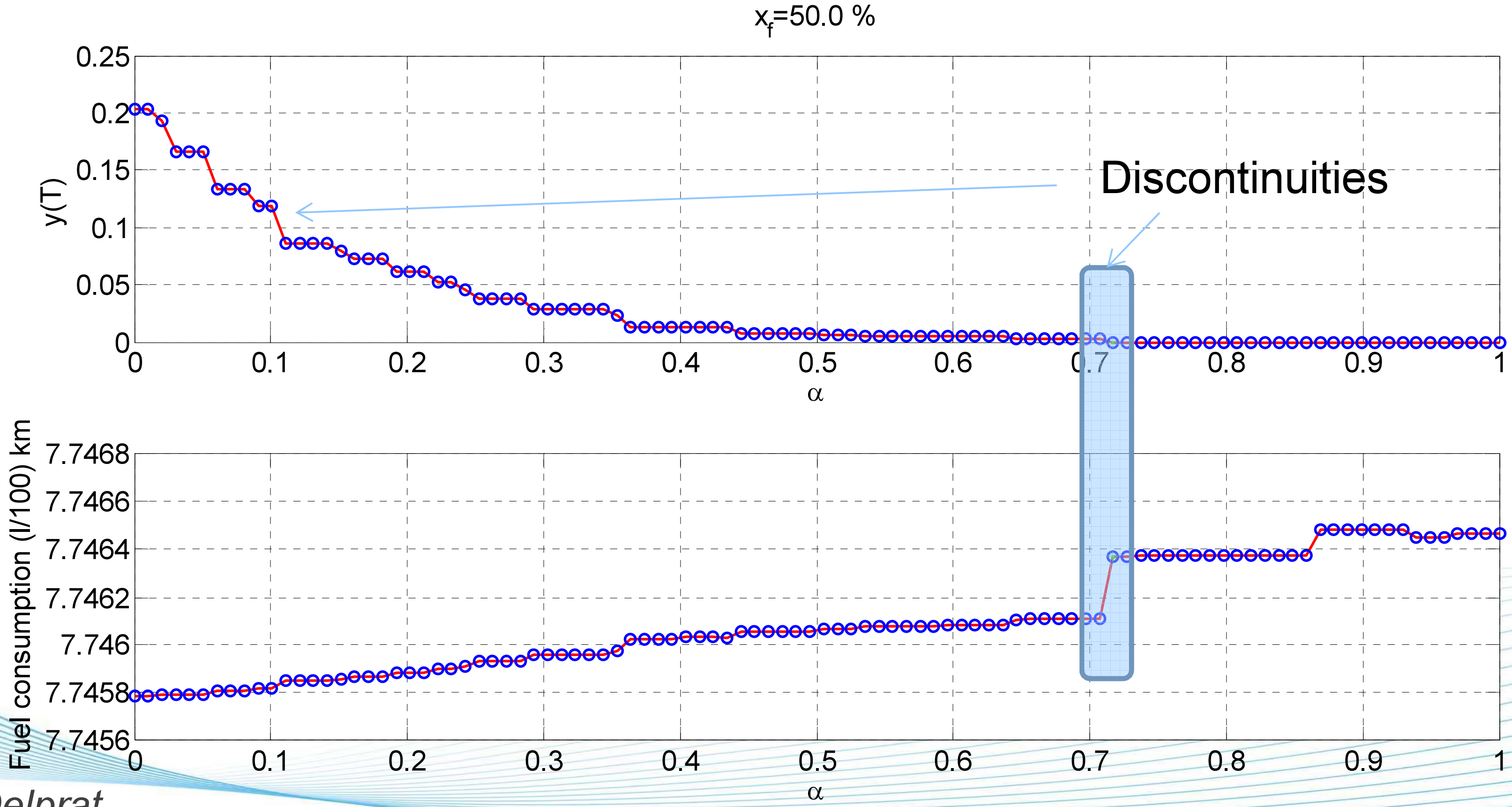
Pontryagin Minimum Principle

State Constraints : Additional dynamics

/D. Kirk 2004/

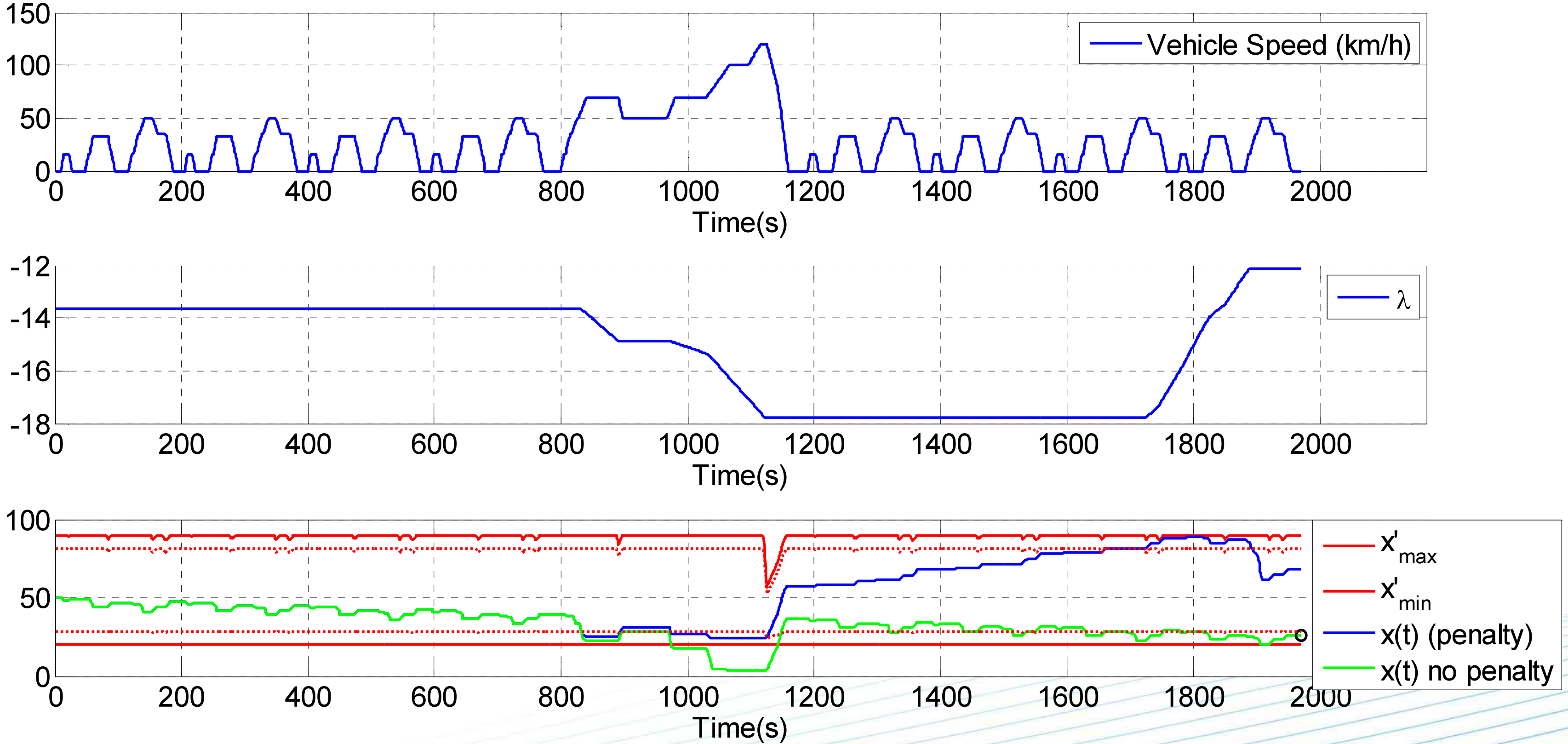
<u>Many problems:</u>

- Implementation : requires careful choice of sampling perdiod



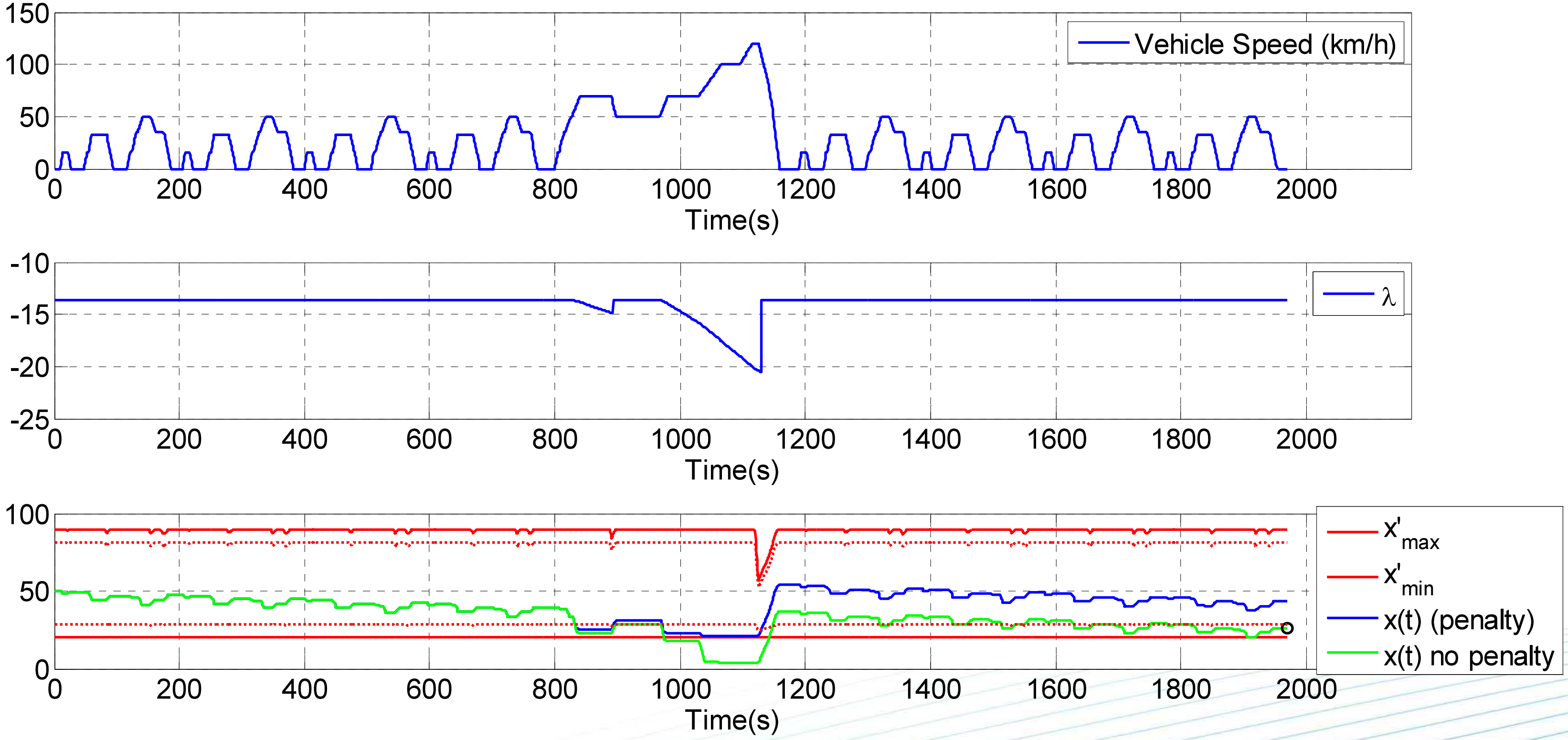
Pontryagin Minimum Principle

Penalty function tuning can be quite tricky!



Pontryagin Minimum Principle

Hint: apply nominal co-state if trajectory is admissible
=> pb: no theoretical support for this



Pontryagin Minimum Principle

State Constraints : Penalty functions

Benefits:

- Implementation is quite simple
- May generate admissible trajectories such that : $x(t) \in [\underline{x}, \overline{x}]$
- May be used in real time

Difficulties:

- Variable co-state: less source code optimization
- Tuning of the Penalty amplitude coefficient α

  => sensitivity to the driving cycle
- Basic idea : increase (decrease) the costate when state reach the boundary

  => Does not allows decreasing (increasing) after
- Solution is NOT optimal with respect to the original problem with $x(t) \in [\underline{x}, \overline{x}]$

Pontryagin Minimum Principle

State Constraints : Theoretical framework

Theoretical framework : /Bonnans & Herman 2006/

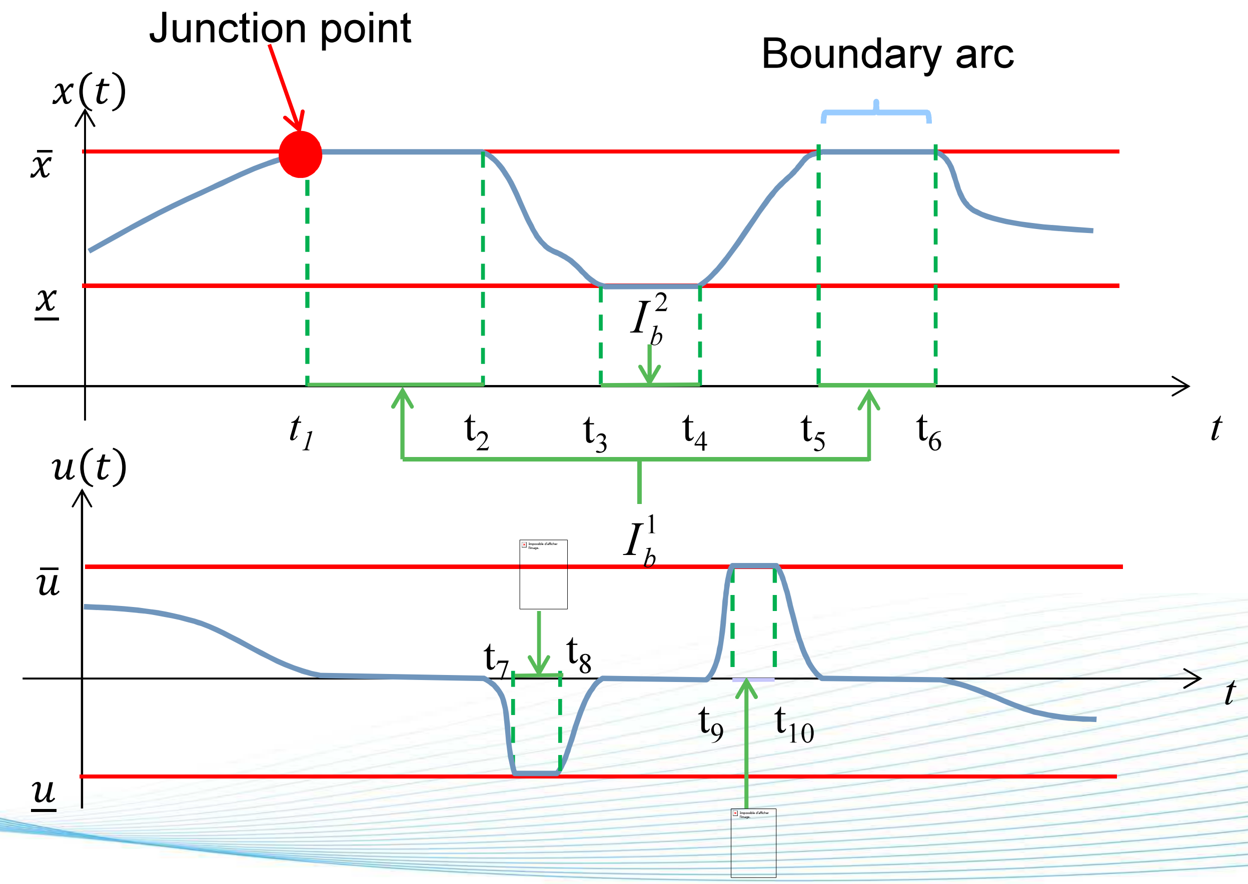
Constraints modeling: $\underline{x} \leq x(t) \leq \overline{x}$ $\underline{u} \leq u(t) \leq \overline{u} \Leftrightarrow g\big(x(t)\big) \leq 0$

$g_0\big(x(t)\big) \leq 0 \Leftrightarrow g_0\big(x(t)\big) = \overline{x} - x(t)$

$g_1\big(x(t)\big) \leq 0 \Leftrightarrow g_1\big(x(t)\big) = x(t) - \underline{x}$

$g_3\big(x(t)\big) \leq 0 \Leftrightarrow g_3\big(x(t)\big) = \overline{u} - u(t)$

$g_4\big(x(t)\big) \leq 0 \Leftrightarrow g_4\big(x(t)\big) = u(t) - \underline{u}$



Pontryagin Minimum Principle

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State Constraints : Theoretical framework

Theoretical framework : /Bonnans & Herman 2006/

$$J = \int_0^T Q\left(x(\tau),u(\tau)\right)\cdot d\tau$$

$$x(0) = x(T) = x_0$$

$$-\dot{\lambda} = H_x^q\left(u,x,\lambda,\eta\right) \text{ on } [0,T]\setminus\mathcal{T}$$

$$\dot{x} = f\left(x,u\right) \text{ on } [0,T]$$

$$0 = H_u^q\left(u,x,\lambda,\eta\right) \text{ on } [0,T]\setminus\mathcal{T}$$

<u>Convexity assumption:</u>

$$H_{uu}^0\left(u,x,\lambda,\eta\right) \geq \alpha$$

$$0 = g_i^{(q_i)}\left(x(t),u(t)\right),\ t\in\mathcal{I}_b^i,\ i\leq r+s$$

$$\eta_i(t) = 0,\ t\in[0,T]\setminus\mathcal{I}_b^i,\ 1\leq i\leq r+s$$

$$0 = g_i^{(q_i)}\left(x(t),u(t)\right),\ t\in\mathcal{I}_b^i,\ i\leq r+s$$

$$0 = g_i^{(q_i)}\left(x(t),u(t)\right),\ t\in\mathcal{I}_b^i,\ i\leq r+s$$

$$H^q\left(u,x,p^q,\eta\right) = Q(x,u) + \lambda.f(x,u) + \sum_{i=1}^{r+s}\eta_i g_i^{(q_i)}$$

Constraints multipliers $\eta_i(t)$
Jump parameters $\upsilon_\tau{}^i$

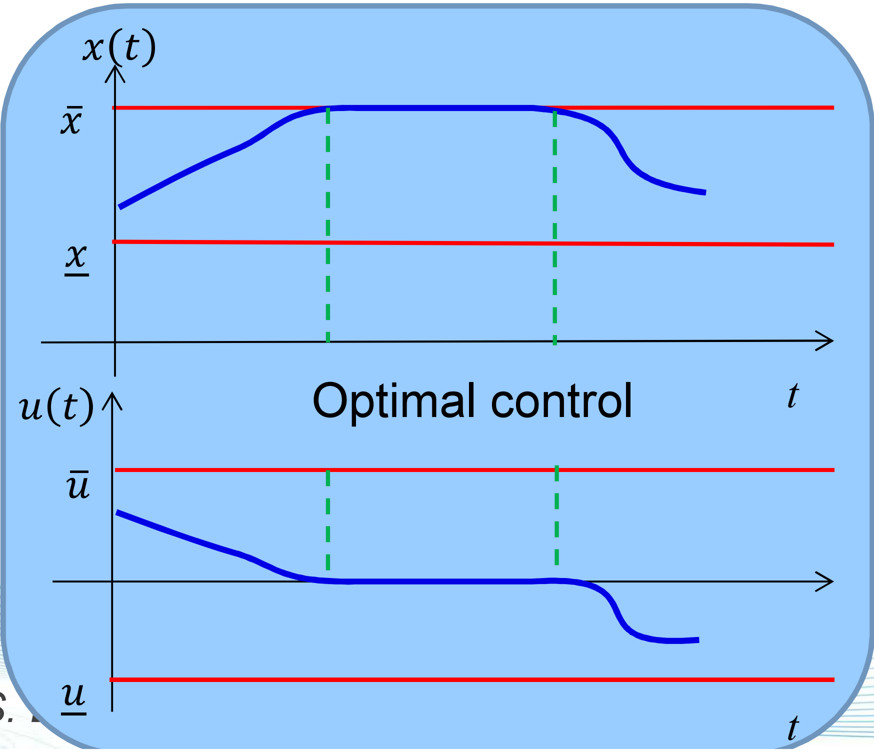
Pontryagin Minimum Principle

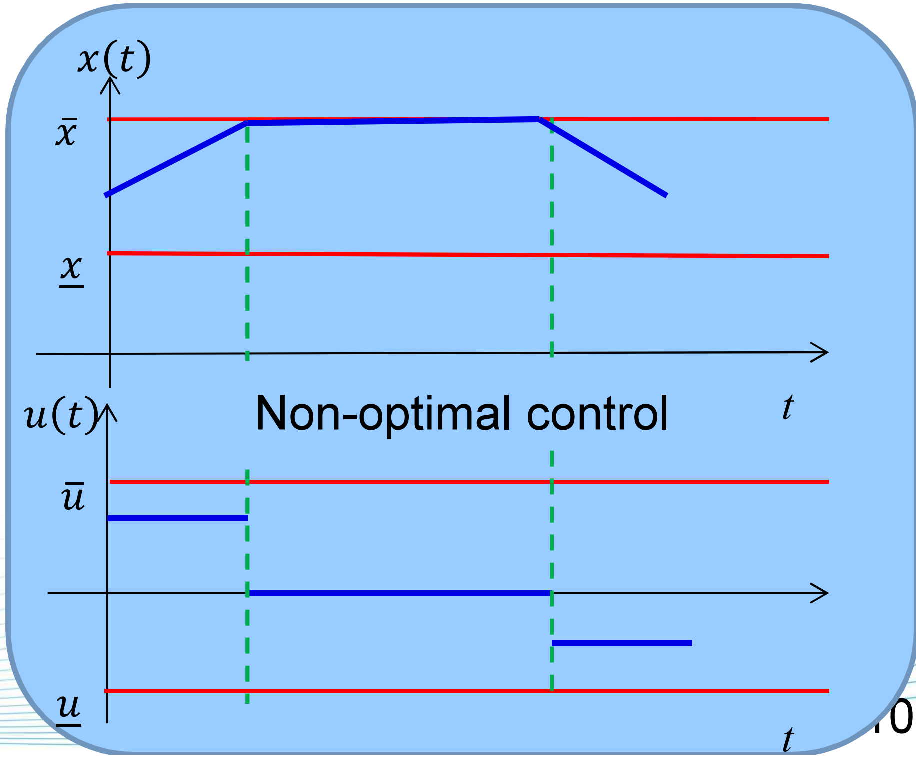
VPPC 2014

State Constraints : Theoretical framework

Condition at the junction points

$$
\left.
\begin{aligned}
&g_i^{(q_i)}\left(u\left(t^-\right),x(t)\right)=0,\ t\in\mathcal{T}_{en}^i,\ i\le r+s\\
&g_i^{(q_i)}\left(u\left(t^+\right),x(t)\right)=0,\ t\in\mathcal{T}_{ex}^i,\ i\le r+s\\
&(-1)^{q_i}\frac{d^{q_i}\eta_i}{dt^{q_i}}(t)\ge 0,\ t\in\mathcal{T}_b^i,\ 1\le i\le r+s
\end{aligned}
\right\}
\qquad
\begin{aligned}
&\frac{dx(t)}{dt}=0\ \forall t\in\mathcal{T}_{en}^i\cup\mathcal{T}_{ex}^i\\
&0=f\left(u,\underline{x},t\right)\qquad 0=f\left(u,\overline{x},t\right)
\end{aligned}
$$





Pontryagin Minimum Principle

State Constraints : Theoretical framework

<u>Important points:</u>

- Co-state is discontinuous at the entry point of a boundary arc (not on the exit)
- From the convex model:
  - State of charge derivative is null at entry and exit point of a boundary arc
  - Control can be obtained at the entry and exit point from the state derivative
  - /Fontaine 2013/=> then the co-state can be also computed

$\Rightarrow$ Optimal control is quite complex to derive

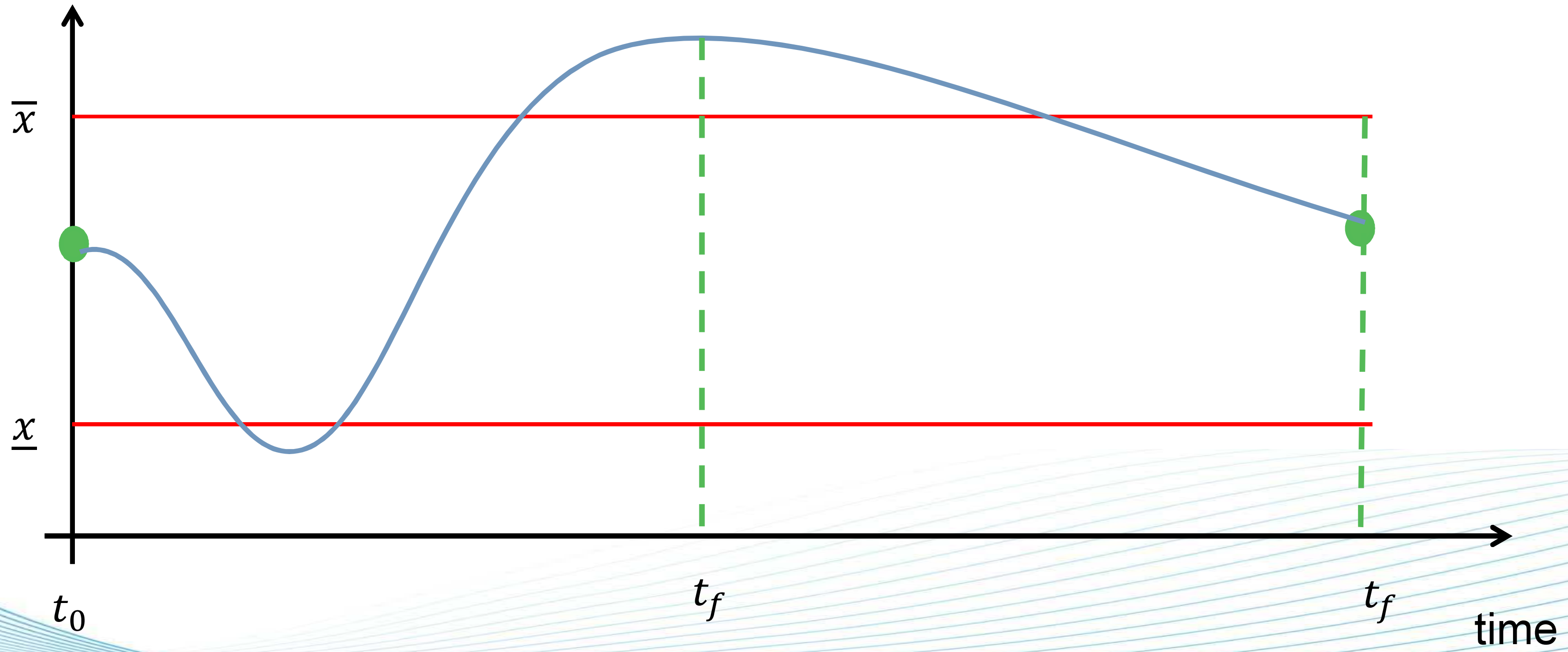
$\Rightarrow$ Iterative algorithms are required

Pontryagin Minimum Principle

State Constraints : Theoretical framework

Algorithm from /Van Keulen 2014/ & /Rousseau 2008/

<u>Step 1 :</u> Locate the maximum error. Update $\left(x_f \in [\underline{x}, \overline{x}], t_f\right)$

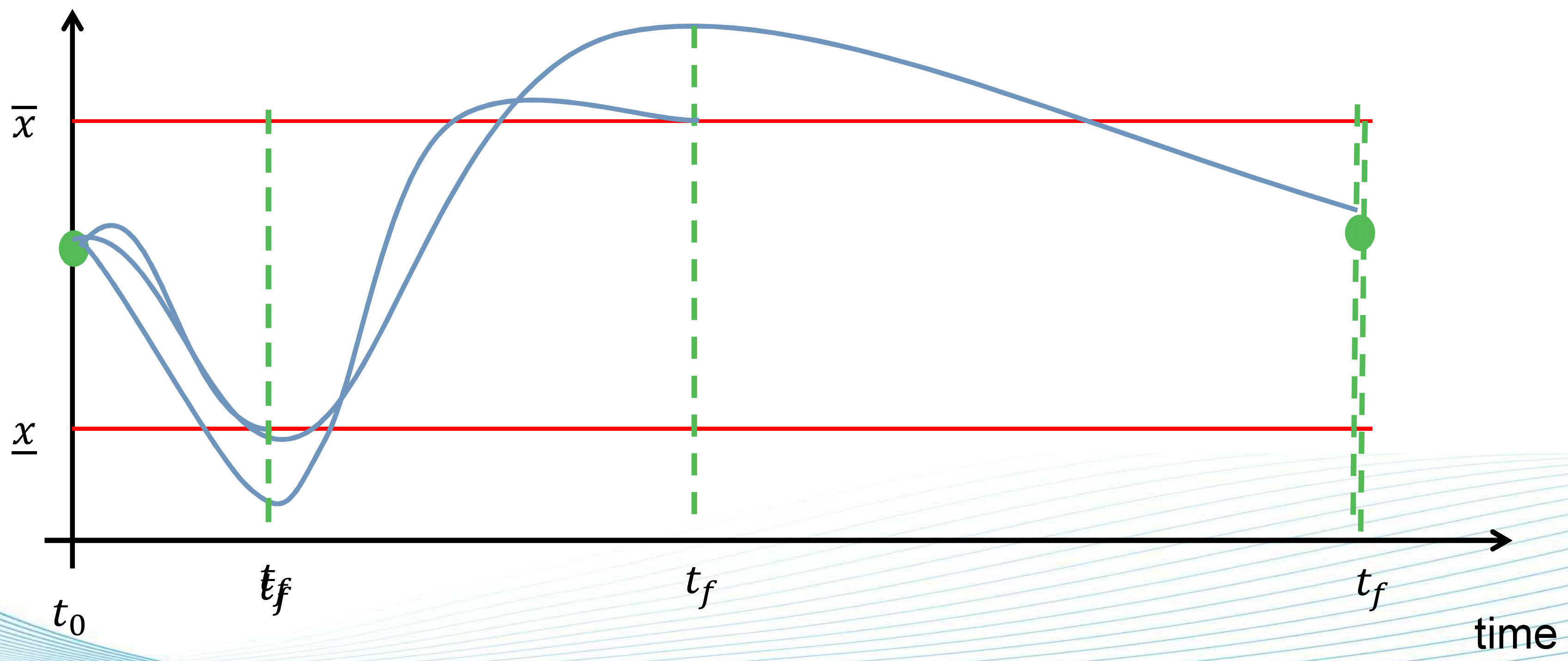


Pontryagin Minimum Principle

State Constraints : Theoretical framework

Algorithm from /Van Keulen 2014/ & /Rousseau 2008/

Step 2 : Iterate until the trajectory is admissible,

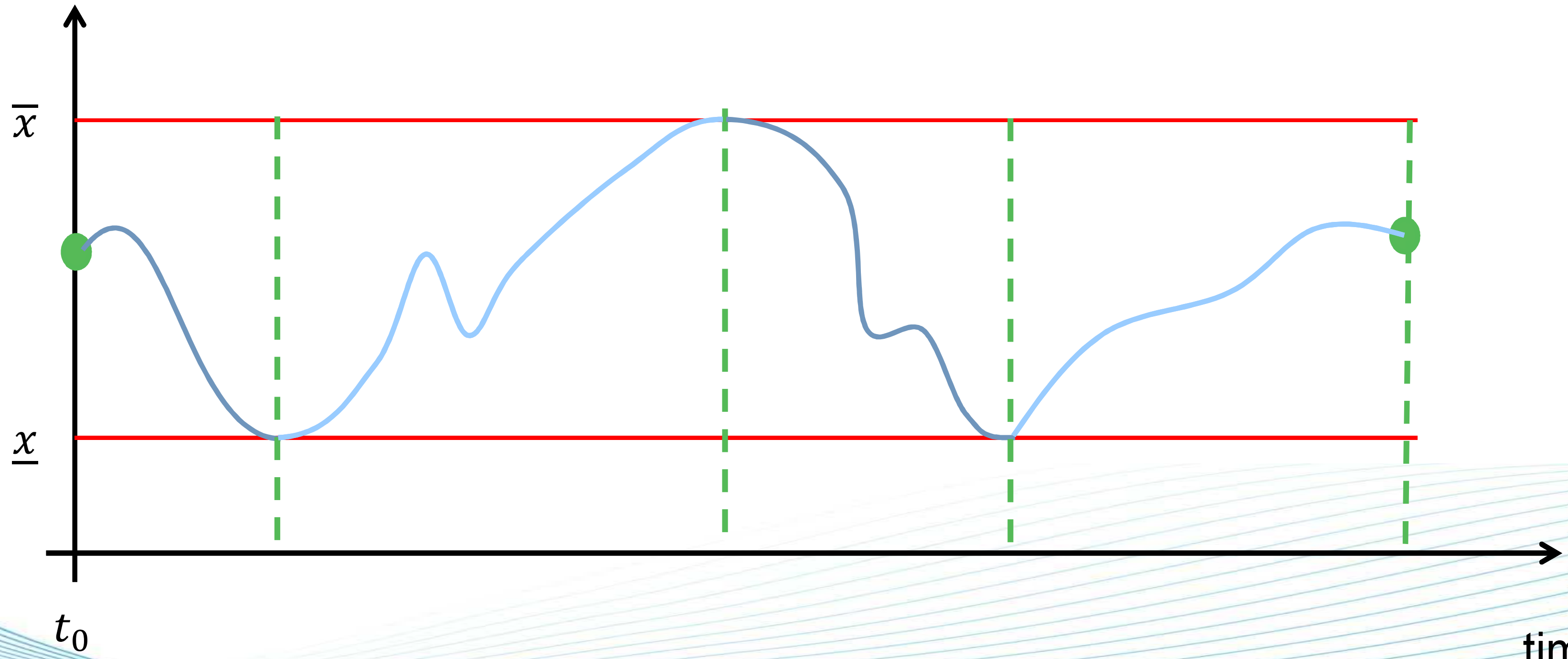


Pontryagin Minimum Principle

State Constraints : Theoretical framework

Algorithm from /Van Keulen 2014/ & /Rousseau 2008/

Step 2 : A part of the optimal trajectory is obtained
=> Iterate to construct the remaining parts of the optimal trajectory



Pontryagin Minimum Principle

State Constraints : Theoretical framework

Algorithm from /Van Keulen 2014/ & /Rousseau 2008/

Benefits:

- Provide an optimal trajectory

Limits:

- Does not explicitly take into account boundary arcs
- Boundary arcs are obtained by iteration, not very efficient algorithm

  => improvement in /Fontaine 2013/

Pontryagin Minimum Principle

Conclusion

- PMP algorithm requires convex models
- Basic version is very simple, efficient
- Integer & binary variables optimization is possible but leads to singular control
- State constraints can be handled
- DP is much straightforward (no need to modify the algorithm to cope with some model's detail)

BUT PMP can be adapted for **real time control**

Pontryagin Minimum Principle

VPPC 2014

A few theoretical open problems

- Non convex models

  No generic results with PMP for non-convex modeling

  In general, approximated solutions can be found using specialized numerical software (e.g. BMI solvers), but no guaranty of convergence

- Pollutants

  If statics maps are assumed to be valid => straightforward

  A few emission & exhaust gas after treatment system models available

  Dynamics measurement of emission is difficult

  Requires a careful modeling : T° model => high order system

- Binary & integer variables

  Reducing the number and frequency of the switches is necessary

  Suboptimal approaches with penalty function => singular variables?

  Need for a "good" mathematical formulation

- Recommended readings

- Guzzella, L., & Sciarretta, A. (2005). Vehicle Propulsion Systems: Introduction to Modeling and Optimization: Springer.
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Part 2 : real time control

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